THE
THEORY
OF THE
WORKING of SHIPS,

Applied to PRACTICE.

CONTAINING
The PRINCIPLES and RULES for SAILING
with the greateſt Advantage poſſible.

By Monſ. *PITOT*,
Of the ROYAL ACADEMY *of* SCIENCES
at PARIS.

Tranſlated from the *FRENCH*,
By *EDMUND STONE*, F.R.S.

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THE

PREFACE.

THE Art of Navigation is divided into two principal Parts, *viz.* the Working Part, and the Guiding or Piloting Part. This latter has from Time to Time been improving, and ſeems to want nothing but the Longitude, to bring it to Perfection. But we cannot ſay this of the former Part, which, though doubtleſs the moſt important of the two, has yet been quite neglected by the ancient Geometricians; it being only in theſe latter Ages, that People began to think of finding out a Theory, to illuſtrate and explain the Practice of Navigators, and make them apprehend the aſſured Principles of an Art, which gives all the Motions to Ships that they are capable of, with the greateſt Expedition and Advantage poſſible.

As nothing can be of more Uſe than a good Way of working a Ship; ſo nothing can be more neceſſary than for Seamen to acquire the Knowledge of thoſe Principles, which may infallibly guide and conduct them in their Operations.

The Advantages they will acquire from this Study, will be, to known how to give the beſt Situations to the Sails and Rudder; ſo as to make the greateſt Way poſſible; moſt readily tack about; exactly find the Lee-Way; and conſequently the true Courſe made good; and the beſt Eſtimate of the Ship's Way: which is all that can be required of the Art, to perform a Voyage, with as much Certainty, Expedition, and, in a Word, Perfection, as poſſible. For, ſince we always know the Place departed from; the Courſe and Diſtance ſailed being known too, will conduce likewiſe to our finding the Place whereat the Ship is. Add to this, that by help of a Perſon expert at the working of a Ship, the Seamen will be enabled to make a good Uſe of the Winds; to get clear of a Coaſt; weather a Cape; avoid Dangers; follow a Ship; give Chaſe to, or avoid her; or get the Advantage of the Wind, and keep it, eſpecially in a Sea-Fight.

It muſt be owned, indeed, we have ſeen very great Seamen, who have been excellently ſkilled in the Art of the working of a Ship, and yet at the ſame Time have known but little or nothing of the Theory of their Art. Amongſt whom are Mr. *Tourville*, *John Bart*, *Du Queſne*, *Gue Trouin*, *&c.* the Superiority of their Genius, long habitual Practice, and their Aſſiduouſneſs in obtaining ſome Advantage from every thing that offered in their daily Practice, did in ſome meaſure ſupply their Ignorance of the Theory. But is it not probable, I may ſay, evidently certain, they would have brought the Art of Navigation to a much higher pitch of Perfection, or at leaſt have ſooner arrived to that heighth of Perfection, for which we admire them, if they had at firſt learned the Theory of their Art.

Amongſt all thoſe who have hitherto acquired ſo much Reputation, there is not one but owes his Knowledge to a very long Courſe of Experience. And therefore, muſt it not be very advantageous to Sea-Officers and Pilots,

lots, to be able to obtain ſuch Helps, without being at the Pains of ſo many Years Experience? A thorough Knowledge of our Theory will render the Practice more familiar and expeditious to them. The Lee-Way will be better known, and the Ship's Rate of going not ſo uncertain: and this Art, which could not be acquired, in any great Degree, but from many Courſes, Voyages, Accidents, and repeated Obſervations, will become the Work of a ſhort Practice, when enlightened and directed by the certain Rules and Principles upon which it is founded.

It is for procuring theſe Eaſements to Seamen, that ſome of the modern Geometricians of great Reputation, have employed their Pains in bringing to Perfection this Part of Navigation, hitherto ſo much neglected. Father *Pardie* firſt undertook to lay down ſome Notions about it, as you will find at the End of a Treatiſe of Local Motion and Moving Forces, publiſhed in the Year 1671. But his Attempts were unſucceſsful. It ſcarcely happening, even in the accurate Sciences, that thoſe who firſt begin to treat of them, do much more than clear the Way for thoſe who come after. The Sagacity and Elegance ſo natural to him, did not prevent his falling into falſe Principles; for all the fundamental

Propoſitions he has endeavoured to lay down, are but ſo many Parallogiſms.

After him came the Chevalier *Renaud*, a Perſon more ſkilful than *Pardie*, in the practical Part of Navigation, who publiſhed a more extenſive Treatiſe upon the Theory of the Working of Ships. But as he has built upon the ſame Foundations as *Pardie*, he likewiſe falls into the ſame Parallogiſms. This Tract, however, made a great Noiſe at firſt, and was ſaid to contain the Principles of a new Science. It was printed in the Year 1689, by Order of *Lewis* the XIVth.

Two or three Years after a little Treatiſe on this Subject was publiſhed by Father *Hoſtte* at *Lyons*, in a Collection of Mathematical Pieces: but he too, treading in the ſame Steps with Father *Pardie*, and the Chevalier *Renaud*, falls into their Errors.

The Chevalier *Renaud*'s principal Error is firſt taken notice of by Mr. *Huygens*. Which learned Geometrician obſerving the Principle from whence the Chevalier *Renaud* determines the different Velocities of a Ship, to be falſe, has thereby overthrown the greateſt part of his Theory. But the Chevalier *Renaud* was not convinced by Mr. *Huygens*'s Objection, and looked upon them only as Difficulties that wanted Explanation and Illuſtration. He anſwered Mr. *Huygens*, to whom Mr. *Huygens* gave a Reply, and the Controverſy increaſed on both Sides.

The Subject in diſpute was very delicate, or elſe Mr. *Bernoulli*, one of the greateſt Geometricians of the Age, would not have taken part in favour of the Chevalier *Renaud*. Though, indeed, ſome Years afterwards he ranged himſelf on the ſide of Mr. *Huygens*; and even went farther, and diſcovered another Miſtake of the Chevalier *Renaud*'s, no leſs ſubtile than the firſt, ſince it eſcaped entirely the penetrating Genius of Mr. *Huygens*; and in reality of no leſs Conſequence, ſeeing it related to the Determination of the Lee-Way, or ſhews how to find the true Courſe of a Ship.

This new Attack of Mr. *Bernoulli*, was as little regarded by the Chevalier *Renaud*, as that of Mr. *Huygens*, he being ſo ſtrongly prejudiced in favour of the Solidity of his own Principles, as not even to be induced to make any Doubt thereof, but obſtinately defended them till his Death, which happening a little time afterwards, put an End to the Diſpute.

Mr. *Bernoulli* finding then that almoſt the whole Theory of the Chevalier *Renaud*, as well as that of the Fathers *Pardie* and *Hoſte*, are neceſſarily overthrown by two Miſtakes, about the Velocity and Lee-Way of a Ship, did himſelf publiſh in the Year 1714, a new Theory of the working of Ships, founded upon undoubted Principles, more ſolidly eſtabliſhed than any before. But whatever Content this Piece may give to Geometricians, the Seamen in general can be as little ſatisfied therewith.

For this Reaſon, therefore, it is, that I determined to write this ſmall Treatiſe, upon the working of Ships, which is ſuitable to the Capacities of Seamen, and wherein the Demonſtrations are more ſimple and ſhort than thoſe of that famous Geometrician, and whoſe Principles are applied to the Figures of Ships nearer approaching to thoſe of ordinary Ships, than his.

The Conſtruction of the Tables, which I have added to this Tract, is another thing of great Conſequence in the Practice of Navigation. They are all ſo contrived, as that Sea-Officers and Pilots may, by bare Inſpection, have the Velocities; and by their Means be enabled to ſolve the moſt important Queſtions or Problems relating to the working of Ships.

Mr. *Bernoulli*, who thoroughly apprehended the Uſe of ſuch Tables, has not been wanting to excite ſomebody to conſtruct them, from the following Account of the Advantages that they will afford to the Practice: "For, ſays he, *page* 96. the great Uſe of "ſuch Tables, will very amply make Amends "for all the Trouble in compoſing them, be-"cauſe by their Means we may be enabled "not only to direct a Ship, ſo as to ſail on a "given Courſe in the beſt manner poſſible, "but alſo immediately reſolve the moſt im-"portant Queſtions relating to that Subject; "ſuch as finding the Point the neareſt to the "Wind that a Ship can ſail upon, and the "beſt Situations of the Keel and Sails, when "ſhe runs from the Wind."

But theſe Tables required by Mr. *Bernoulli*, make but a ſmall part of thoſe annexed to our Treatiſe; as will eaſily appear to any one who will be at the Pains to examine them.

This Treatiſe is divided into Nine Sections. The firſt eight contain the Theory and Principles of the working of Ships; and the ninth ſhews the Uſe of the Tables, together with the Reſolution of the chief Problems in the Art.

In the firſt Section are laid down the general Principles, or Laws of the Impulſe of Fluids. The ſecond treats of the Principles peculiar to the Art, ſuch as Definitions and Explanations of what is meant when we ſay a Ship runs before the Wind, large, or upon the Wind, *&c.* or when ſhe keeps or runs to Windward; or falls to Leeward.

In the third Section we ſhew how to find the moſt advantageous Situations of the Sails and Keel of a Ship, with reſpect to the Direction of the Wind, as well when ſhe runs upon the Wind, as goes large: when there is no Lee-Way, or the Reſiſtance the Ship finds in cutting the Water with her Head, or Prow, is infinitely ſmall, with regard to the Reſiſtance her Side would find in cutting the Water.

Although the Method purſued by us in this Section, differs from that of Mr. *Bernoulli*, yet we have obtained the ſame Equation, or Literal Expreſſion as his; though, indeed, it is Mr. *Huygens*'s, who gave it without a Demonſtration: but Mr. *Bernoulli* was the firſt who generouſly communicated to the Public the Analyſis thereof, as may be ſeen in the fifth Chapter of his Theory.

In the fourth Section are laid down the Situations of the Sails, and the Drift, or Lee-Way; when the horizontal Sections of Ships are taken for right-lined Polygons of any number of equal Sides, making unequal Angles with one another. The greater the Number of theſe equal Sides are, the nearer will the ſaid horizontal Sections approach to thoſe of the real Shapes of Ships, and the nearer will what is deduced from them approach to the Truth. So that by this Section the Quantity of the Lee-Way may be determined as near as you pleaſe, and that after the moſt certain and natural manner, conſidering the Irregularity of the Curvatures of the Ships. But ſince that Method of Approximation is very long and troubleſome, we have introduced into our fifth Section, the ſame Suppoſitions as Mr. *Bernoulli*, who conſiders the horizontal Sections of the Hull, as made up of two equal Segments of a Circle joined together by a common Chord, repreſenting the Direction of the Keel

The Theory of the Impulſe of Fluids upon curve Surfaces, is one of the moſt important and uſeful Applications of the modern Method of Fluxions. By this it is, that ſome of the greateſt Geometricians have found out the curve Line, which moving about a right Line, generates a curve Superficies, to which, if that Part of a Ship within the Water, be ſhaped, ſhe will move with the leaſt Reſiſtance poſſible.

But a Ship of this moſt advantageous Shape, is ſuppoſed to be deſtitute of Lee-Way; that is, ſhe runs always in the Direction of her

Keel, which can never happen, unleſs when ſhe ſails directly before the Wind.

In the ſixth Section we have ſhewn how to find the Ratio of the Velocities of a Ship, under different Situations of her Sails, when her Lee-Way is different, as likewiſe the Velocities, not only with regard to the Angles made by the Sails and Keel, but alſo when the Angles of Incidence of the Wind falling upon the Sails, the Forces or relative Velocities of the Wind, and the Quantities of Sails, are all different.

Since Mr. *Bernoulli* in the *Leipſick* Tranſactions, Diary of the Learned, and in the 14th Chapter of his Theory, has determined the Curvature of Sails when filled by the Wind, I ſhall not mention any thing about that here. I only take from him the Method of finding the mean Effort of the Wind upon the Sails, ſo that the Effect of the Curvature of the Sails may be reduced to the bare Suppoſition of flat Sails, as has been already obſerved by the Chevalier *Renaud*.

As to the beſt Situation of the Maſts and Yards, you may conſult the learned Treatiſe of Mr. *Bouguer* upon the Maſting of Ships, which gained the Prize offered by the Academy in the Year 1727.

In the ſeventh Section, is given the Theory of the Rudder. The Chevalier *Renaud* is the firſt who, I believe, determined the beſt Angle the Tiller of a Rudder muſt make with the Ship's Keel, ſo as to moſt readily bring her about; being indeed the ſame that Father *Hoſtte*, Mr. *Huygens*, *Parent*, and *Guinée*, found out to be the beſt Angle for the Sails of our common Wind-mills, Mr. *Bernoulli* in a ſmall Corollary deduced the beſt Poſition of the Angle of the Sails, but we determine the ſame Angle when the Lee-Way is conſidered, when others hitherto ſuppoſed it to be nothing.

Becauſe the working of Oars has a great Affinity to that of the Rudder, we have given the Theory of the former in the eighth Section, and ſhewn the Proportion that the Parts of the Oar ought to have, ſo as the Force of the Rowers may be applied with the greateſt Advantage.

In the ninth Section, you have a Reſolution of the moſt important Queſtions relating to the working of Ships. And as the eight firſt Sections chiefly regard the Theory, ſo this here is entirely deſigned for Practice. All the Queſtions therein being deduced from the Principles laid down in the Theory, without any Uſe of Algebra; that thereby it may better ſuit the Capacities of Seamen. In this

Section we begin with the Conſtruction and Uſe of the practical Tables, and point out ſome eſſential Faults that are often committed in Practice.

Suppoſe a Ship be about to depart or ſet ſail, and the Angle of her Courſe with the Line of the Wind be known; then the firſt Problem ſhews how to find the Situation of the Sails, and the Quantity of the Lee-Way.

The ſecond and third Problems determine the Ratio of the Velocities or Ways run by Ships; the fourth ſhews how to find the relative Velocity of the Wind at Sea; the fifth and ſixth, how to find the Ratio of the different Velocities of one or more Ships, in all the various Poſitions of the Sails, Directions and Forces of the Wind, poſſible.

The ſeventh Problem ſhews how to find out and compute, the Advantage ariſing from ſetting the Sails according to thoſe Angles marked down in our Tables, which are taken for the moſt advantageous. The eighth compares the Velocities or Ways made by two Ships, when one of them runs large, and the other upon the Wind.

The ninth Problem ſhews the manner of computing how much a Ship falls to Leeward, when ſhe goes large, and how much ſhe gets to Windward, when ſhe goes near the Wind.

When two Ships or two Fleets are ſtriving to obtain the Weather-Gage, and in order thereto, one of them keeps as cloſe as poſſible up to the Wind, whilſt the other runs with her Sails and Keel diſpoſed in the moſt advantageous Manner poſſible. Now the tenth Problem ſhews what Advantage this latter has over the former. As a Ship ſometimes in foul Weather is obliged to lie with her Broadſide to the Wind, the eleventh Problem ſhews nearly what Way ſhe makes in that Situation.

As a Ship ſometimes in foul Weather is obliged to lie with her Broad-ſide to the Wind; the Eleventh Problem ſhews nearly what Way ſhe makes in that Situation.

Since Ships do not always carry the ſame Quantity of Sails, the twelfth Problem lays down Rules whereby to find out and compute the Ways run by a Ship, according as ſhe carries more or leſs Sails.

Laſtly, the thirteenth Problem ſhews the Manner of comparing the Ways run by the ſame Ship, carrying more or leſs Sails, when ſhe goes before the Wind, goes large, and goes upon the Wind.

Beſides the Solution of theſe Problems, which are illuſtrated with ſeveral Examples we have laid down many other general Rules, to find the abſolute Reſiſtance of the Water upon the Prow and Body of the Ship, the Reſiſtance of the Wind upon the Sails, and the abſolute Velocity of the Wind.

An Extract from the Regiſter *of the* Royal Academy of SCIENCES, *of the* 14*th of* February, 1731.

Mr. *Nicole*, and the Abbot *Bragelonne*, who were nominated to examine a Treatiſe of Mr. *Pitot*, concerning the working of Ships, wherein, (after having given Rules to find their Lee-Way and Velocities, both when the Halves of all the horizontal Sections, at unequal Diſtances from the Keel, are ſuppoſed to be ſimilar Polygons of any number of equal Sides, making unequal Angles with one another, and when every one of thoſe half Sections is a Segment of a Circle of any number of Degrees) he has chiefly aimed at reducing to Practice, the Theory of this part of Navigation, publiſhed by Mr. *Ber-*

Bernoulli, in the Year 1714; having made their Report thereof: The ACADEMY are of Opinion, that this Work, eſpecially the Tables made by Mr. *Pitot*, which may be uſed by any one of a ſlender Skill in Mathematics, cannot but be very uſeful, not only for directing Ships in their Courſe, but alſo expeditiouſly reſolving the moſt important Queſtions about the working of them, founded upon ſuch Suppoſitions as can cauſe but very ſmall Errors. In Conſequence whereof I have ſigned the preſent Certificate. Dated at *Paris*, the 18th of *February*, 1731.

FONTENELL,

Perpetual Secretary of the
Royal Academy of Sciences.

THE ELEMENTS OF THE WORKING of SHIPS.

SECT. I.

Of General Principles.

1. WHEN a Surface receives the Stroke or Impulſe of a Fluid, we may indifferently either conſider the Fluid to move againſt the Surface, or the Surface againſt the Fluid, or elſe the one and the other to each partly partake of the reſpective Velocity, wherewith the Surface receives the Fluids Impulſe.

2. The Magnitude of the Strokes of a Fluid againſt the ſame Surface with different Velocities are in the duplicate Ratio of theſe Velocities. This is

a general received Principle, being agreeable to Experience, and it is demonſtrated commonly from hence, that when the Fluid has more Velocity, every little part of the Fluid ſtrikes with more Force in the Ratio of its Velocity, and in the ſame Time there are a greater Number of Parts in the ſame Ratio too, which ſtrike, from whence ariſes a duplicate Ratio. This Demonſtration in reality is only Phyſical, but we have laid down (in the Memoirs of the Academy for the Year 1725, p. 91.) a geometrical Demonſtration of it, according to *Galileo*'s Hypotheſis of the Fall of Bodies.

3. The oblique Strokes of a Fluid againſt the ſame Surface, are in the duplicate Ratios of the Sines of the Angles of Incidence.

Fig. 1. If A B be a Surface Oblique to the Direction R B of a Fluid, the perpendicular A P to that Direction will be the Sine of the Incidence, A B being taken for the Radius. Now it is evident that the Number of the Parts of the Fluid that would ſtrike the Surface, if it were perpendicular, is to the Number of the Parts that ſtrike it obliquely, as A B to A P. But moreover it is a Maxim received and demonſtrated, that the perpendicular Stroke of each Part of the Fluid, is to their Oblique Strokes, as the Radius, is to the Sine of the Incidence of the Obliquity, or as A B is to A P. which make a duplicate Ratio.

4. If the ſame Fluid meets ſeveral unequal Surfaces under the ſame Angle of Incidence, the Magnitudes of the Strokes upon or againſt each Surface, will be in the ſingle Ratio of the Magnitude of the

Surfaces. This Principle has no need of a Demonſtration.

5. From the Principles aforeſaid it follows, that if a Fluid Strikes unequal Surfaces with unequal Velocities, the Magnitudes of the Blows will be in the ſingle Ratio of the Surfaces, and the duplicate Ratio of the Velocities.

6. If a Fluid with the ſame Velocity ſtrikes againſt unequal Surfaces, and under different Angles of Incidence; the Magnitudes of the Strokes ſhall be in a Ratio compounded of the ſingle Ratio of the Surfaces, and the duplicate Ratio of the Sines of Incidence.

7. But if equal Surfaces are ſtruck under unequal Angles by a Fluid with unequal Velocities; the Magnitudes of the Strokes in this Caſe will be in a Ratio compounded of the duplicate Ratio of the Velocities, and the duplicate Ratio of the Sines of the Angles of Incidence.

8. Laſtly, if unequal Surfaces are ſtruck by a Fluid with unequal Velocities and unequal Angles of Incidence; the Magnitude of the Strokes upon each Surface, will be in a Ratio compounded of the ſingle Ratio of the Surfaces, the duplicate Ratio of the Velocities, and the duplicate Ratio of the Sines of the Angles of Incidence.

9. Under whatever Angle of Incidence a plain Surface receives the Impulſe of a Fluid, the Line along which that Surface tends, when puſhed by the Fluid, is always perpendicular to that Surface, as in whatever Direction C D, C E, C F, the Surface A B may be puſhed, its Tendency will be al- Fig. 2.

ways along the Line C G, which is perpendicular to A B.

Fig. 1.

10. The relative Magnitudes of the Strokes of a Fluid againſt ſeveral Surfaces, as of the Wind upon ſeveral Plane-Sails may be expreſſed by Lines: If for Example, the Line A B be taken for the Expreſſion of the whole or perpendicular Force of the Wind againſt the Sail, and if A P be the Sine of the Angle of the Incidence of the Direction of the Wind againſt the ſame Sail; then ſhall A Q be the Expreſſion of the relative Force of the Wind. for (by *Art.* 3.) the Magnitude of the perpendicular Impulſe upon the Sail A B, is to the Magnitude of the oblique Impulſe, as the Square of the Radius, is to the Square of the Sine of the Angle of Incidence, or as $\overline{AB}^2$ is to $\overline{AP}^2$. But becauſe of the ſimilar Triangle A B P, A P Q; it will be $\div$ A B, A P, A Q and A B: A Q :: $\overline{AB}^2$ $\overline{AP}^2$ wherefore, &c.

11. If ſeveral plane Surfaces differently ſituated, making invariable Angles with each other, do at the ſame time receive the Impulſe of a Fluid; every Surface will have its particular Tendency, or will be diſpoſed to move in a different Direction. But we can always by the Theory of Compound Motions, find a middle Tendency equally parting the Efforts of each, and by that Means reduce the Sum of the Impreſſions or Efforts made upon all the different Surfaces to one Surface only, which is perpendicular to the Line of the middle or mean Tendency, and this we call the *Mean Line* of the moving Force.

12. But a Curve Surface may be always conſidered as made up of an infinite Number of ſmall plane Surfaces, and every one of theſe ſmall Planes has its particular Tendency, or endeavours to move in a different Direction; conſequently a Curve Surface has an infinite Number of different Tendencies, amongſt which there is a mean Tendency, which we call the Line of the mean Tendency. So that it follows from hence that when one wants to find the mean Tendency, along which a Curve Surface is puſhed by a Fluid, that Surface is to be looked upon as a Plane, upon which (by *Art.* 9.) the Line of the mean Tendency, muſt always be perpendicular. Hence, firſt we may conſider Sails as Plane Surfaces; and ſecondly, reduce the ſaid particular mean Tendency of ſeveral Sails to one mean Tendency only, equally parting the Efforts or Impreſſions of the Wind made on different Sides upon different Sails.

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SECT II.

Of the Motion of a Ship and the chief Particulars of the Working of her.

THAT the Knowledge of the Principals of the working of a Ship may be made eaſy, let H M be the Keel of a Ship, M the Prow, H the Poop, Fig. 3. A B the Line of the Direction that the Wind Blows in, which we always call the Courſe of the Wind, or Line of the Wind, and C D the Plane or Flat Sail, to which we ſuppoſe all the reſt are reduced (by *Art.* 11 and 12). and the perpendicular B G to this Sail, will be the Line of the moving Force.

13. When a Ship at Reſt is ſet in Motion by the Action of the Wind upon the Sails, her Velocity or Way will encreaſe by little and little, until the Action of the Water upon her Prow and Body, becomes equal to that of the Wind upon the Sails, and ſince Action is directly contrary and equal to Reaction, the Direction in which ſhe is drove back by the Water which ſhe meets, muſt be always the ſame as the Line of the moving Force, and this is the chief Point of the whole Theory of the

Working of a Ship, that is, the mean Reſiſtance of the Water againſt the Ship muſt be equal and directly oppoſite to the mean Force of the Wind upon the Sails, whereby the Axes of Equilibrium of the Reſiſtance of the Water, and the Force of the Wind directly correſponds in a right Line.

14. But altho' a Veſſel be ſtruck in the Direction of the moving Force B G, yet nevertheleſs ſhe cannot move and ſhape her Courſe along that ſame Direction, but when B G parts each way equally the Efforts of the Water upon the Veſſel, and this can never happen but when the Line of the moving Force is parallel to the Keel, or the Ship Sails before the Wind. The Figure of all Veſſels being ſuch, as to be divided always into two Parts, equal and ſimilar by the Keel.

15. In order for a Ship always to ſhape her Courſe in the Line of the moving Force B G, her Figure ought to be ſuch, that the Line of the moving Force may every where find an equal Reſiſtance as ſhe cuts the Water; but this is only peculiar to a Cylindrick or ſpherical Figure.

16. If the Reſiſtance that a Ship finds in cutting the Water with her Head or Prow M, were infinitely ſmall with reſpect to her Reſiſtance in cutting the Water with her Side; ſhe would make her way always in the Direction of the Keel H M.

17. But that Reſiſtance which the Ship ſuffers by cutting the Water with her Head, being neither nothing, nor infinitely ſmall with reſpect to the Reſiſtance ſuffered by cutting the Water with

her

her Side, the Ship puſhed in the Direction B G muſt run along a Line B L, lying between B G and B M.

18. The common Practice of Navigators is to direct and ſet the Keel in the Line along which they would have the Ship ſhould move, or ſhape her Courſe; and this they do by carrying the Prow or Head of the Ship always in that Line by means of the Helm. But the greater the Angle M B G made by the Keel, and the Line of the moving Force is, the more does the Line B L of the Ships Courſe, deviate from the Keel, or from the Line in which the Ship would run, whereby the Angle M B L becomes ſo much the greater. That Angle is called the Angle of the Drift or Lee-way *.

19. In all Caſes of ſailing, the Angle A B M of the Line of the Wind A B, and the Keel B M, or the Courſe to be ſteered, is given or known always. Now the principal thing in the working of a Ship is firſt to know for every Angle A B M, the poſition of the Sail C D and from thence determine the Direction of the mean Line of the moving Force B G. Secondly from the Angle M B G, to find the Line B L of the Ships Courſe, and then the Lee-way Angle M B L: after which all the reſt of the Angles will be known; that is, the Angle A B D of the Winds Courſe and the Sails

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* *The Drift or Lee-way of a Ship, is the Oblique or Slanting Courſe ſhe really runs, being meaſured by the Angle formed by this real Courſe, and that ſhe ought to Steer, or by the Angle which her Wake makes with the Keel.*

Sails. The Angle D B M of the Sails and the Keel. The Angle A B L of the Line of the Wind, and that in which the Ship runs; and if neceſſary, the Angle A B G of the Line of the Wind and the Line of the moving Force. The Angle D B L of the Sails, and the Ships Courſe, and laſtly the Angle L B G of the Ships Courſe, and that of the moving Force.

20. When the Angle A B M of the Line of the Wind, and the Keel is equal to two right Angles or thereabouts, we ſay that the Ship runs before the Wind. If that ſame Angle be a right Angle, ſhe runs with the Wind upon the Beam. If it be obtuſe, ſhe goes large, and if acute, goes upon a Wind. There are alſo ſome other Diſtinctions to make hereafter, which we ſhall explain in the uſe of our Tables.

21. If there be four Ships repreſented by the Numbers 1, 2, 3, and 4, and thro' the firſt a Line B R S B be drawn perpendicular to the Line of the Wind A B, the Ships 1 and 2 will be each equally to Windward, the third Ship will be to Windward of the firſt by the Diſtance B R, and the fourth to Leeward of the firſt by the Length B S: ſo that the Ship 3 will be more to Windward than the reſt. Fig. 4.

22. When the Angle of the Wind's Courſe, and the Ship's true Courſe is a right Angle; the Ship always is ſaid to keep her Wind. When that Angle is acute, ſhe goes upon the Wind, or gets upon the Wind; and when it is obtuſe, ſhe falls to Leeward, or runs from the Wind. So if we Fig. 4.

ſuppoſe in the four Ships that the Courſe be according to the Direction of the Keel, the fourth Ship will keep her Wind, becauſe the Angle A B M is a right Angle; the firſt will get to Windward, ſince the Angle A B M is acute; and the ſecond and third will run or fall to the Leeward, becauſe that ſame Angle is obtuſe.

23. Amongſt all the Angles A B M, which may be taken in ſailing upon a Wind, or to Windward, there is always one of more Advantage than the reſt; for it is evident that the nearer the Ship keeps up to the Wind, the Angle A B M is the more acute; but going beyond a certain Point, if that Angle were to be made more acute, or the Wind kept up to ſtill nearer, a very conſiderable Diſadvantage would follow, chiefly for theſe two Reaſons. Firſt, the Sail B D being between the Line the Wind, and the Keel, or in the Angle A B M, the Angle A B D becomes too acute, and the Action of the Wind upon the Sail but very ſmall, the Strokes being (by the *Art.*) as the Squares of the Sines of the Angles of Incidence, which muſt very much leſſen the Ships way. Secondly the more acute the Angle D B M of the Sail and Keel is, the nearer does the Angle M B G of the Keel and the Line of the moving Force approach to a right Angle, one of them being always the Complement of the other. Now it is evident that the nearer the Angle M B G approaches to a right Angle, the more is the Ship drove ſideways: which may increaſe her Lee-way, or the Angle M B L, to ſuch a Degree as to cauſe her to fall off, and ſo

loſe more by the Lee-way, than can be obtained by leſſening the Angle A B M.

24. Hence ſince the more the Angle A B M is leſſen'd, the more that of the Lee-way M B L is increaſed; it follows, that there muſt be a Point of Equality between the Diminution of the one, and the Augmentation of the other: That is, if the former be leſſened one Degree at that Point, the ſecond will be increaſed one Degree, which ought to be well obſerved for this is that which determines the ſmalleſt Angle which the Ship's Courſe ought to make with that of the Wind.

25. Of all the Directions that can be given to the Keel, with reſpect to the Courſe or Line of the Wind, when a Ship looſes Wind or runs to Leeward, it is evident the moſt advantageous is that being the ſame with the Line of the Wind; the Angle A B M being equal to two right Angles, and the Ship then running directly before the Wind. But altho' it be really true that the Sails of the hindmoſt Maſts of Ships with ſeveral Maſts, ſailing directly before the Wind, do hinder the Wind from falling upon the Sails of the foremoſt Maſts, which is the reaſon that a ſide Wind is more advantageous than a direct Wind; becauſe the former fills all the Sails: Yet that Objection is not at all contrary to the Principles and Rules of the working of a Ship. For in this Caſe we ought to have regard only to thoſe Sails expoſed to the Action of the Wind, and conſider that when the Ship ſails directly before the Wind the motive Force driving the Ship forwards is leſs

than that ariſing from a Side-wind, becauſe ſhe carries leſs Sails.

26. The Angle A B M, or the Direction of the Keel with reſpect to the Line of the Wind, being known or given; I ſay, the Poſition of the Sail C D is not then to be taken at pleaſure, but from that ſome Angle A B D, or D B M may be obtained, being the moſt advantageous poſſible for the Ship's Way; for the ſmaller the Angle M B G is, the more does the Direction of the moving Force approach to that of the Keel. Now the Ship makes ſo much the greater Way, the nearer the Line of the moving Force approaches to the Point of the leaſt Reſiſtance ariſing, by the cutting the Water, or according to the Direction of her Keel. But when the Angle M B G is leſſened, the Angle D B A is leſſened too, becauſe the Angle D B G is always a right Angle, and by leſſening the Angle D B A, the motive Force, or Action of the Wind upon the Sail, is leſſened; becauſe the Sine of the Angle of Incidence is leſſened. There is therefore a middle Point, whereat the Angle A B D of the Wind's Courſe with the Sail is of the greateſt Advantage for the Ship's Motion. And this we ſhall ſhew how to find, in the Caſe of a Ship ſhaping her Courſe upon or near the Wind; the other Caſe, when ſhe Sails large, following of courſe. And ſo I now proceed to the Inveſtigation of theſe Angles of the greateſt Advantage, firſt ſuppoſing, with Mr. *Huygens* and *Bernoulli*, the Lee-way to be nothing, or the Reſiſtance the Ship ſuffers by cutting the Water with

her Head, to be nothing, or infinitely ſmall, with reſpect to her Reſiſtance ariſing from cutting the Water with her Side. And this I ſhall do much after the ſame manner as I have laid down in the *Memoirs of the Academy* for the Year 1727.

SECT. III.

Of the moſt Advantageous Situations of the Keel, and the Sail of a Ship, both when ſhe ſails upon or near the Wind, and runs from the Wind, or Large.

27. IN order to find the beſt Situation of the Sail C D, when the Line of the Ship's Courſe or Keel is in the Direction B K F, (*Fig.* 6, & 7.) it is evident, firſt, that A B being the Line of the Wind, the Ship in *Fig.* 6. will ſail upon or near the Wind, and that of *Fig.* 7. will ſail large. Secondly, that if the Diſtance B E be taken for the Radius, E P will be the Sine of the Angle of the Incidence of the Wind upon the Sail. Thirdly, that having drawn B G perpendicular to the Sail, and G K perpendicular to the Direction B F, if B G expreſſes the Effort wherewith the Ship is puſhed in the Direction B G of the Line of the moving Force, B K will expreſs the lateral Force, whereby ſhe is puſhed in the Direction B F of the Courſe, or Keel: ſo that of all the Angles ABD, the beſt is that which gives the greateſt lateral Force, expreſſed by B K. The Queſtion then is

brought to this, *viz.* to determine the Value of B K, and then find the *Maximum* thereof by the Method of Fluxions. To do which, having deſcribed the Quadrant of a Circle T B S, about the Point E, with the Diſtance E B, as a Radius, draw E N parallel to B F, to which draw B V perpendicular, as alſo E Q: alſo draw N M perpendicular to the Line S E F, the Perpendicular to the Line of the Wind.

28. Let the Radius B E be called a, the Sine of Incidence P E, x, and M E, M N (which are given) b, and c, reſpectively: then (by *Art.* 3.) the perpendicular Impulſe of the Wind upon the Sail, is to its oblique Impulſe as EB^2 or $\overline{ES}^2$ is to $\overline{EP}^2 :: aa : xx :: a : \frac{xx}{a}$. So that when a repreſents the whole perpendicular Impulſe of the Wind upon the Sails, $\frac{xx}{a}$ will be that of the oblique Force, or the Value of B G. Now becauſe of the ſimilar Triangles N M E, B P F, it will be N M (c) : M E (b) :: B P $\left(\sqrt{aa-xx}-x\right)$: P F $\left(\frac{b}{c}\sqrt{aa-xx}-x\right.$. Therefore in the Caſe of *Fig.* 6. when the Ship goes upon the Wind, E F is $= \frac{b}{c}\sqrt{aa-xx}-x$. And in the Caſe of *Fig.* 7. when ſhe goes Large, E F will be $= \frac{b}{c}\sqrt{aa-xx}+x$. But the ſimilar Triangles E M N, E Q F, give E N (a) : N M (c) :: E F $\left(\frac{b}{c}\sqrt{aa-xx}\pm x\right.$ in both Caſes$\left.\right)$: E Q $= \frac{a}{b}\sqrt{aa-xx} \pm \frac{cx}{x}$. Moreover becauſe of the ſimilar Triangles B E V, or B E Q, and B G K, it

will be EB (a) : EQ $\left(\frac{b}{a}\sqrt{aa-xx} \pm \frac{cx}{a}\right)$ :: BG $\left(\frac{xx}{a}\right)$: BK $= \frac{bxx\sqrt{aa-xx} \pm \frac{cx^3}{a}}{a^3}$, being the Value of the lateral Force, parallel to the Ship's Courſe in both Caſes, *viz.* $\frac{bxx\sqrt{aa-xx}-cx^3}{a^3}$, in the firſt Caſe, and $\frac{bxx\sqrt{aa-xx}+cx^3}{a^3}$, in the latter; the *Maximum* of which muſt be found. wherefore in the firſt Caſe, taking the Fluxion of $\frac{bxx}{a^3}\sqrt{aa-xx}-cx^3$, which is $\frac{2baax\dot{x}-3bx^3\dot{x}}{a^3\sqrt{aa-xx}}$, and making it equal to nothing, we ſhall obtain this Equation $2aab-3bxx-3cx\sqrt{aa-xx}=0$, which being freed from Surds, and aa put for $bb+cc$, and the whole divided by aa, will be brought down to this $x^4 \begin{array}{l} -\frac{1}{3}aaxx \\ -\frac{1}{3}bbxx \end{array} = -\frac{4}{9}aabb$. In the ſecond Caſe, if we take the Fluxion of $\frac{bxx}{a^3}\sqrt{aa-xx}+cx^3$, and work as before; we ſhall obtain the ſame Equation as in the firſt Caſe.

29. Therefore this Equation contains the *Maximum* of thoſe two Caſes; and indeed theſe two Roots $xx = \frac{1}{2}aa + \frac{1}{6}bb \pm \frac{1}{2}\sqrt{a^4 - \frac{10}{9}aabb + \frac{1}{9}b^4}$, are both affirmative real ones, *viz.* $xx = \frac{1}{2}aa + \frac{1}{6}bb - \frac{1}{2}\sqrt{a^4 - \frac{10}{9}aabb + \frac{1}{9}b^4}$, for the Sine of the Incidence of the moſt advantageous Angle made by the Line of the Wind and Sails, and

conſequently that of the Sails and Keel, when the Ship runs upon a Wind, or to windward, and $xx = \frac{1}{2}aa + \frac{1}{8}bb + \frac{1}{2}\sqrt{a^4 - \frac{10}{9}aabb + \frac{1}{9}b^4}$, for the Sine of the Incidence of the beſt Angle, when the Ship ſails large.

30. Becauſe the Angle ABF or ABQ, is always equal to the Angle MNE, whoſe Sine is $ME = b$: therefore to find out or compute the moſt advantageous Angle of the Sails, and the Line of the Wind, that of the Keel and this Line being of any number of Degrees, whoſe Sine found in the Tables ſhall be the Value of b; we muſt put the Value of b and its Powers, as alſo the Values of the Radius a, or Sine total, and its Powers, into the Expreſſion $xx = \frac{1}{2}aa + \frac{1}{6}bb$, &c. and then we ſhall have the Value of xx; and the ſquare Root thereof, will be the Sine of the Incidence of the beſt Angle of the Sails and Line of the Wind. If for Example, the Angle of the Line of the Wind, and the Keel be 55 Degrees for the firſt Caſe, and 125 for the ſecond. In both Caſes we ſhall have $a = 100000$, and $b = 81915$; and putting the Powers of theſe Values into the Expreſſions of the Roots of both Caſes, and compleating the Computation, which is much longer than difficult to do, in the firſt Caſe we ſhall get an Angle of 35 Degrees and 25 Minutes for that of the Line of the Wind and Sails, which being taken away from 55 Degrees, and the Remainder 19 Degrees 35 Minutes will be the beſt Angle of the Sails and Keel; and in the ſecond Caſe, the

Angle of the Line of the Wind and Keel will be found to be 70 Degrees and 26 Minutes, and the beſt Angle of the Sails and Keel 54 Degrees and 34 Minutes. And after this manner is it that we have computed our third Table for Angles of every 3 Degrees made by the Line of the Wind and the Rhumb Line, or Ship's Courſe, both when ſhe goes upon a Wind, and goes large, which is ſufficient for the Practice.

31. I might have gone further, to the finding the beſt Angle made by the Keel and Line of the Wind, when the poſition of the Sail or Angle it makes with the Keel, is given: but ſince by help of our Tables, when one of theſe Angles is given the other is immediately found, I ſhall not ſtay to do this; Brevity being my Deſign.

32. If the Direction of the Keel be perpendicular to the Line of the Wind, the Ship will then receive the Wind equally, or goes with the Wind upon the Beam, and then will B F be parallel to E F, the Points N and M falling upon S, and $b=a$. So that to obtain in this Caſe the beſt ſituation of the Sails, we muſt put a for b in the Equation $xx = \frac{1}{2}aa + \frac{1}{6}bb \pm \frac{1}{2}\sqrt{a^4 - \frac{10}{9}aa\,bb + \frac{1}{9}b^4}$. and then will xx be $= \frac{2}{3}aa$, and $x = a\sqrt{\frac{2}{3}}$. for the Sine of the Angle of Incidence A B D.

33. To find the beſt Direction of the Keel for the Ship to go upon the Wind, in the moſt ſimple Caſe, *viz.* when the Plane of the Sails is parallel to the Keel, or the Angle E B Q vaniſhes or becomes nothing. The whole Magnitude B G of the Winds impulſe upon the Sail B Q (for ſuppoſe for a Moment B Q to coincide with B D) being divided into two lateral Forces B K, K G, it evi-

dently appears that the Ship can only go in the Direction B Q F, when plying to Windward, with the lateral Force B K; for the lateral Force K G is directly oppoſite to it. Therefore, in this Caſe, the Inclination of the Sail, or the Direction of the Keel B Q F, muſt be the ſame as that of the Sail in the Article aforegoing, *viz.* the Sine A B F muſt be $a\sqrt{\frac{2}{3}}$ or 54 deg. 44 min.

34. Now leaving the Keel in the ſaid moſt advantageous Situation for plying to Windward, if that of the Sail be at the ſame time wanted for getting the two moſt advantageous Situations both of the Keel and Sail, when the Ship plies to Windward, it eaſily appears that here b is $= \sqrt{\frac{2aa}{3}}$; and ſo we need but put $\sqrt{\frac{2}{3}aa}$ for b in the Equation

$$x = \sqrt{\frac{1}{2}aa + \frac{1}{8}bb \pm \frac{1}{2}\sqrt{a^4 - \frac{10}{9}aabb + b^4}},$$

and then will x be $= \sqrt{\frac{11}{18}aa \pm \frac{5}{18}aa}$, or $x = \sqrt{\frac{1}{3}aa}$, and $x = \sqrt{\frac{8}{9}aa}$. But it is evident that it is the leaſt Root $x = \sqrt{\frac{1}{3}aa}$, which is the Sine of the moſt advantageous Angle made by the Line of the Wind and the Sail; becauſe that ſame Angle muſt be always leſs than that of the Line of the Wind, and the Keel, and on the contrary, the greater Root $x = \sqrt{\frac{8}{9}aa}$ will give an Angle greater. Nevertheleſs this greater Root is not uſeleſs; for it is the Sine of the moſt advantageous Angle of the Sail, and the Line of the Wind, when the Ships runs large; the Sine of the Line of the Wind and the Direction of the Keel being $\sqrt{\frac{2}{3}aa}$:

SECT. IV.

Of the Situations of the Sail with regard to the different Courſes and Lee-ways of Ships, when the Planes of the horizontal Sections of their Hulls are right lined Polygons.

35. LET the Profil of the Body of a Ship be repreſented by Fig. the 8th, and ſuppoſe B M H E to be the Part immerged in the Water; then is this Part terminated by the Water Line B E, call'd the Hull. Now it is evident that this Hull may be looked upon as made up of an infinite Number of Planes or thin horizontal Slices, lying one upon another, whoſe Circuits, will be the ſame with thoſe of the Ships planking anſwerable to each Slice: we ſhall therefore in this Section take every Slice for a right lined Polygon, which we ſhall eſteem as the Figure of the Ship itſelf. Theſe Polygons altho' irregular, will have each of their Halves perfectly equal and ſimilar, ſeparated by a Line equally dividing the Ship into two Parts from the Prow to the Poop. For Example, let the Polygon M H Q q. *Fig.* 12. be an horizontal Section or Slice of the Ship, the Line M H, which may be taken for the Keel, divides her into two Parts equal and alike.

Fig. 9. 36. When a Plane Surface M R receives obliquely the Impulſe of a Fluid in any Direction L R P, L P R. it is evident, firſt, that if M R be taken for the Radius; M P will be the Sine of the Angle of Incidence. Secondly, that (*Art.* 9.) the Direction according to which the Surface is preſs'd by the Fluid, is always perpendicular to that Surface. Thirdly, that the Expreſſion of the Force by which the Surface is puſh'd by the Action of the Fluid in the Direction R I, perpendicular to the Surface, may be divided into two lateral Forces; the one in the Direction R K, perpendicular to the Direction of the Fluid, and the other in the Direction R l, parallel to the ſaid Direction. But (by *Art.* 10.) If M R expreſſes the Quantity of the Fluids perpendicular Impulſe againſt the Surface; M Q will be the Quantity, of the oblique Impulſe, according to the Angle of Incidence M R P. Therefore taking R I perpendicular to the Surface and equal to M Q; if this Force R I be divided into two other lateral ones R K, R l perpendicular and parallel to the Fluids Direction; R K will expreſs the perpendicular, and R l the parallel Effort. Then making M R $= a$, and M P $= x$, P R will be $= \sqrt{aa-xx}$, and becauſe of the ſimilar Triangles M R P, M P Q and R I K, we ſhall have M R (a) : M P (x) : : M P (x) : M Q $= \frac{xx}{a} =$ R I; and M R (a) : M P (x) : : M Q or R I $\left(\frac{xx}{a}\right)$: K I or K l $= \frac{x^3}{aa}$ expreſſing the parallel lateral Force. In like manner M R (a) : P R $\left(\sqrt{aa-xx}\right)$: : R I $\left(\frac{xx}{a}\right)$: R K $= \frac{xx\sqrt{aa-xx}}{aa}$ being the perpendicular lateral Force. Therefore for every Angle of Incidence we ſhall have the expreſſions of the three Forces, *viz.* $\frac{xx}{a}$ for that of the whole Force; $\frac{x^3}{aa}$ that of the lateral parallel Force; and $\frac{xx\sqrt{aa-xx}}{aa}$ for that of the perpendicular lateral Force.

37. Hence, when ſeveral unequal Surfaces receive the Impulſe of a Fluid under equal Angles of Incidence, the Quantities of the lateral Impreſſions perpendicular, and parallel to the Direction of the Fluid, are to each other in the ſingle Ratio of the Magnitude of thoſe Surfaces.

38. And when the Angles of Incidence are unequal, the parallel lateral Forces are to one another, in a Ratio compounded of the ſingle Ratio of the Surfaces, and the triplicate Ratio of the Sines of Incidence; and the perpendicular lateral Forces are to one another, in a Ratio compounded of the ſingle Ratio of the Surfaces, the Duplicate of the Sines of Incidence, and the ſingle Ratio of the Sines of their Complements.

Fig. 10. 39. Let there be two equal Surfaces, M R N, M r n, making a conſtant Angle with one another, ſtruck by a Fluid in the Direction M L, ſo that the Angles of Incidence M R V, M r n be equal: this being ſuppoſed, it is evident, firſt, that the whole Force R I of the Impulſe upon the Surface M R N is equal to the whole Force r i of the Impulſe upon the Surface M r n: ſecondly, the parallel lateral Force R l, is equal to the parallel one r l: and thirdly, the perpendicular lateral one R K, equal to the perpendicular lateral one r k. But the two lateral perpendicular ones R K, R K, r k, being directly oppoſite, do make an Equilibrium, and mutually deſtroy one another; ſo that the two Surfaces are not puſhed in the Direction of the Fluid, but with the Sum of the two parallel Efforts R l, r l.

40. If the Fluid meets two Surfaces under unequal Angles of Incidence, the whole Forces R I, r i, being reſolved into two lateral Forces R K, R l; r k, r l, perpendicular and parallel to the Direction of the Fluid B L; it is evident that the Sine of the Incidence M V being greater than the Sine of the Incidence M u, the perpendicular lateral one R K, muſt be greater than the correſpondent one r k; and conſequently, the two Surfaces are puſhed in the Direction, parallel to that of the Fluid, with a Force expreſſed by the Exceſs of R K above r k, or by R K — r k, during the Time that they are puſhed in the Direction of the Fluid, with a Force expreſſed by the Sum of the lateral parallel ones R l, r l; therefore to get the Direction and Effort compounded of the two lateral Efforts, we muſt take upon L B (continued out) B F equal to R l + r l, and the perpendicular B E equal to R K — r k: then will the Diagonal B O of the Parallelogram B F O E, be the Expreſſion of the compound Force, wherewith the two Surfaces are puſhed by the Fluid in the Direction G B O. Fig. 11.

41. All that we have ſaid here, with regard to the Force of a Fluid moving againſt the Surfaces M R N, M r n, at reſt, is of itſelf applicable to the Reſiſtances ſuffered by the ſaid Surfaces, *&c.*

when cutting the Water, as they move on, in the Direction B L; ſo that if the ſaid Surfaces repreſent the Prow or Fore-part of a Ship, in ſhape of a Rhombus, whoſe Keel is H M; B L will be the Line of the Ship's Way, B G that of its moving Force, its Perpendicular D C, the Poſition of the Sail; and the Angle M B L, the Lee-Way Angle: all this is manifeſt (by *Art.* 9, 14, 17, *&c.*) and may be moreover applied eaſily to Ships whoſe horizontal Sections are Parallelograms. But it is much better to paſs on to conſider the Figures nearer approaching thoſe of ordinary Ships.

Fig. 12. 42. To determine the Situation of the Sail with regard to the Courſe and Lee-Way of Ships, whoſe horizontal Sections of the Hull, are right-lined Polygons, as M Q H q; I conſider, firſt, that (*Art.* 34.) the Keel H M does always divide the Ship, or the Polygon, into two Parts, equal and alike. That the Side M N = M n, likewiſe N P = n p, P Q = p q; and if through the Points R, S, T, and r, s, t, the Middles of each Side, the Lines T Y, S X, R V, and r u, s x, be drawn parallel to the Direction of the Courſe B L, the Angles P T Y, N S X, M R V, and M r u, n s x, will be the Angles of Incidence of the Action of the Water upon each Side; ſo that having (as in *Art.* 3. aforegoing) divided the whole Force into two lateral ones, perpendicular and parallel to the Direction of the Courſe B L the Ship ſteers, and taken B F equal to the Sum of the parallel lateral Forces upon all the Sides M N, N P, P Q, and M n, n p, and then B E equal to the Sum of the perpendicular lateral ones upon the Sides MN, NP, PQ, leſs the Sum of the perpendicular lateral ones upon the Sides Mn, np; the Diagonal OB, ſhall be (as in the Article aforegoing) the Direction and Magnitude of the mean Force of the Water upon the Prow or Fore-part of the Ship, which muſt be equal to, and directly oppoſite to the Force of the Wind upon the Sail. And therefore OBG will be the Line of the moving Force, and its Perpendieular DBC the Poſition of the Sail, or Yard, and MBL the Angle of the Lee-Way.

43. If the Angle MBL be greater than the Angle BMN, it eaſily appears that there is but one half MNPQ of the Ship, upon which the Reſiſtance of the Water exerts its Effects; and ſo having drawn the Lines RV, SX, TY, ru, sx, ty, through the Points R, S, T, r, s, t, the Middles of each Side, parallel to the Ship's Courſe BL; the Angles MRV, NSX, PTY, Mru, nsx, and rty, will be the Angles of Incidence, under which the Sides of the Polygon do receive the Impulſe of the Water, and having divided the whole Force of the Action of the Water upon each Side into two lateral ones, perpendicular and parallel to its Direction, or the Courſe BL, we take BF equal to the Sum of the parallel lateral Forces, and the perpendicular BE, equal to the Sum of the perpendicular lateral ones; for it is plain, that in this Caſe there is not one of the perpendicular Efforts oppoſite to each other, becauſe the Action of the Water will have no Effect, but upon one half of the Ship.

44. The Angles M N P, N P Q, made by the Sides of the Polygon, being known; to compute eaſily the Value of the Angle L B G, for every Angle of the Lee-Way M B L, we have made the firſt Table of entire Forces, and lateral ones, perpendicular and parallel to the Direction of a Fluid, for every Angle of Incidence to every thirty Minutes; which is ſufficient enough for Practice. This Table has many other Uſes, which we ſhall not here explain, as being foreign to our Deſign. Its Conſtruction is as follows:

1. We take Unity for the Magnitude of the Surface. 2. We ſuppoſe alſo the Velocity of the Fluid, or the Surface moved in the Fluid, to be 1. 3. The perpendicular Impulſe of the Fluid upon the Surface 1, with the Velocity 1, to be 20000. all theſe for Reaſons hereafter explained. Now having found (by *Art.* 36.) when a is the Radius, and x the Sine of the Angle of Incidence, that if a expreſſes the whole Perpendicular Impulſe, $\frac{xx}{a}$ will be the oblique Impulſe; $\frac{x^3}{aa}$ the parallel lateral one, and $\frac{xxy}{aa}$ or $\frac{xx}{aa}\sqrt{aa-xx}$, the perpendicular lateral one: therefore to obtain the parallel and perpendicular lateral Forces, the Magnitude of the whole Impulſe, perpendicular to the Surface, being 20000, we ſay, as $a : \frac{x^3}{aa} :: 20000 : \frac{x^3 \times 20000}{a^3}$. Which will be equal to the parallel lateral one, and $a : \frac{xxy}{aa} :: 20000 : \frac{xxy \times 20000}{a^3} =$

perpendicular lateral one. If the Angle of Incidence, for Example, be 15 Degrees, and the Computation be made by the Logarithms; we ſhall have $x = 941300$. Wherefore $xx = 1882600$, and $xx \times 20000 = 2312703$, by adding the Logarithm of 20000, which is 430103; and finally if from the Logarithm of $xx \times 20000$, which is 2312703, the Logarithm of aa, the ſquare of the Radius, which is 2000000, be taken away; we ſhall have the Logarithm of $\frac{xx \times 20000}{aa} =$ 312703; to which the Number 1340 anſwers in the Tables: ſo that this Number is the Value of the Magnitude of the whole Impulſe, when the Angle of Incidence is 15 Degrees. To obtain the Logarithm of $\frac{x^3 \times 20000}{a^3}$; to the Logarithm of $\frac{xx \times 20000}{aa}$ I add the Logarithm of x, and afterwards take away the Logarithm of a from the Sum; and then we ſhall have the Logarithm of $\frac{x^3 \times 20000}{a^3} = 254003$, the Number anſwering to which is 346, being the Value of the lateral parallel Force. Laſtly, to obtain the Logarithm of $\frac{xx}{a^3} \times \sqrt{aa - xx} \times 20000$; to the Logarithm of $\frac{xx \times 20000}{aa}$ I add the Logarithm of $\sqrt{aa - xx}$ (being the Co-ſine) which is 998494, and from the Sum ſubtract the Logarithm of a; then will the Remainder be the Logarithm of $\frac{xx \times \sqrt{aa - xx} \times 20000}{a^3}$ $= 311197$, whoſe correſpondent Number 1294, is the Value of the perpendicular lateral Force. And after this manner we conſtruct our firſt Table.

Fig. 12. 45. We ſhall now apply the Manner of uſing the Tables, in the following Example. Having drawn the Line M V X, *&c.* perpendicular to the Line B L of the Ship's Way, and continued out the Line M N to e, N P to f, and M n to g; if the Angle N M B be 30 Degrees, the Angles M N P, N P Q, M n p, *&c.* each 165 Degrees, and the Angle M B L 6 Degrees; the Angle B M V will be 84 Degrees, and the Angle R M V 54 Degrees. Conſequently the Angle of Incidence M R V 36 Degrees, equal to the Angle M e X. But the Angle S N e, the Supplement of the Angle M N P, is 15 Degrees, which being taken from the Angle N e X = 36, and there will remain the Angle of Incidence N e S upon the Side N P, 21 Degrees, equal to the Angle S f Y, becauſe of the Parallels S X, f Y: and to obtain the Angle of Incidence P T Y upon the Side P Q, we ſubtract the Angle T P f of 15 Degrees from the Angle P f Y of 21°, and we ſhall have the Angle P T Y of 6 Degrees. The Angle M r u may be found by taking the Angle B M r = 30° from the Angle B M u = 96°, and the Remainder will give the Angle r M u of 66°, whoſe Complement is the Angle of Incidence M r u of 24°, equal to the Angle M g x, from which having ſubtracted the Angle S n g = 15°; there will remain the Angle of Incidence n s x, upon the Side n p, of 9 Degrees. Thus, having found all the Angles of Incidence for every Side, we aſſume in the Table the Values of the lateral Forces, both parallel and perpendicular, and diſpoſe all after the following manner.

Sides.	The Angle of Incidence.	Lateral parallel Forces.	Lateral perpendicular Forces.
M N	36°	4161	5590
N P	21	921	2398
P Q	6	23	217
M N	24	1345	3023
N P	9	76	483

Now it is plain (by *Art.* 40.) that having taken B F equal to the Sum of all the lateral Forces which are parallel, being here 6526, and B E = 4699; the Difference between the Sum of the perpendicular lateral Forces of the Sides M N, N P, P Q, and the Sum of the ſame perpendicular lateral Forces of the Sides M n, n p, will form the Parallelogram B E O F, whoſe Diagonal B O will give the Direction and Magnitude of the mean Force of the Water; and laſtly, to obtain the Angle L B G of the Rhumb-Line, and that of the moving Force, being equal to the Angle F B O. In the right-angled Triangle B F O, we ſay, as B F 6526 is to F O 4699, ſo is the Radius 100000, to the Tangent of the Angle F B O 72004. whoſe correſpondent Angle is 35° 45′. ſo that the Angle

MDG will be 41° 45′, and the Angle MBD of the Sail and Keel 48° 15′.

46. When the Figure of a Ship is ſuch as her horizontal Sections may be taken for right-lined Polygons, it is then after this Way we find the Situations of the Sail for every Angle of the Lee-Way, and make a Table, wherein, by knowing the Situation of the Sail, we may from thence obtain the Courſe and the Lee-Way, or contrariwiſe.

47. But though Ships are made of very different Shapes, there are none whoſe horizontal Sections are right-lined Figures. For Curves have almoſt always been preferred, and that for two principal Reaſons. Firſt, the Strokes of Fluids upon curve Surſaces are much leſs than upon plane ones of the ſame Extenſion, as will be ſeen in the following Section; and ſo by bending the Prow, and even all the Planking of Ships into Curves, they ſuffer much leſs Reſiſtance in paſſing through the Water, than if they conſiſted of plane Surfaces; an Advantage which all thoſe who firſt built Ships found out by Experience, or, I may ſay, by the Light of Nature, which directs every Man to judge of ſeveral mechanical Effects without computing them: and ſecondly, it is an Advantage, as we know by Geometry, that curve Surfaces take up more Space, or have more Capacity than plane Surfaces.

48. Although the Theory of the Impulſe of a Fluid upon curve Lines be a part of Geometry, applied to Mechanicks, the moſt profound and abſtract; this Difficulty might be overcome, if the

Curves given to Ships were regular, geometrical, or mechanical: but ſince Shipwrights make Ships of ſuch Shapes and Curvatures as they themſelves think moſt convenient, without either the Knowledge of the Nature or Properties of thoſe Curves, which are almoſt always irregular, and ever different in different Ships. And as thoſe Irregularities deprive us of a geometrical Exactneſs, yet we can ſo nearly obtain the Situations of the Sails, the Courſe, and Lee-Way, as not to cauſe any ſenſible Difference in Practice; and that by inſcribing Polygons in thoſe Curves, or treating them as regular Curves nearly ſimilar to them.

49. In Ships of the common Form, we are to conceive two principal Curves, the one vertical, and the other horizontal. The former will be obtained by ſuppoſing the Sides of the Ship to be cut by an upright Plane perpendicular to the Keel, and the latter, by ſuppoſing her cut by a horizontal Plane. From whence it follows, that the horizontal Sections of the Hull are unequal, and grow leſs and leſs, the nearer they are to the Keel or Bottom of the Ship. For Example, let a g f K be the Plane of the horizontal Section of a Ship, cut at the Height A F, b l e n, that of the horizontal Section at the Height B E, and c o d p the horizontal Section at the Height C D, *&c.* now all theſe Planes, or Slices, may be taken for Curves, or ſimilar Figures, without any ſenſible Error, although they may be a ſmall Matter different: nor need any Regard be had to the Curve of the upright Section, perpendicular to the Keel. For firſt, Fig. 8.

to obtain the Courſe, the Lee-Way, and the Poſition of the Sails, there is no Occaſion to know the abſolute Magnitude of the Impulſe of the Water againſt the Prow and Body of the Ship. Becauſe our Buſineſs is only to get the mean Direction of the Reſiſtance of the Water. And ſince all the Sections are ſimilar, it is very manifeſt that the Relation between the Reſiſtances of the Water made on both Sides the Rhumb-Line, will be the ſame for all. And conſequently, the Tendency of the mean Reſiſtances of all the Sections will all be upon the ſame Line, and make but one and the ſame Tendency, as if all the Sections were equal, and ſimilar, immediately placed upon one another.

If we cannot ſuppoſe, without ſome ſenſible Error, the horizontal Sections of the Hull to be ſimilar Planes; then may we make Computations of the ſame Tendencies for ſeveral Sections taken at different Diſtances from the Keel; and then it will be eaſy to find the mean Tendencies between all thoſe that have been found for the ſame Angle of the Lee-Way at every Section. By which Means, we may approach as near as deſired to the exact Tendencies belonging to the different irregular Figures of Ships. And then may make a Table of the Lee-Ways and Velocities of every Ship; as ſhall be more largely explained hereafter: their Conſtruction will, indeed, appear troubleſome; but whatever this may be, it is far ſhort of their great Uſe.

50. Although Mr. *Bernoulli* ſeems not to have any Regard to the Curvature of Ships, taken according to their vertical Sections perpendicular to the Keel, there is all the Reaſon in the World to believe he tacitly ſuppoſed it, by taking all the horizontal Sections for Planes, or ſimilar Figures. But the Circuits of his Sections being not as we have ſuppoſed, regular Curves, they may be referred to ſome regular Curve, or even ſome Segments of the ſame Curve; we have referred them with Mr. *Bernoulli*, to Segments of Circles, which are taken of ſome Number of Degrees, more or leſs, according to the Curvity of different Ships. Our Tables are calculated for Segments of 20 Degrees to 60 Degrees, running on for every 5 Degrees, to which the different Curvatures of common Veſſels correſpond. But if there be any Ship whoſe Curvature cannot be referred to any one of the Segments of a Circle from 20 Degrees to 60 Degrees. In this Caſe a Polygon may be inſcribed in that Curve (as we have already mentioned) and then we may proceed according to the Way laid down in the 44th and 45th Articles; it being at the ſame Time evident, that the greater the Number of the Sides of the Polygon is, the nearer ſhall we come to the true Determination; and thus may we approach as near as neceſſary.

SECT. V.

Of the Reſiſtances and mean Tendencies of curve-lined Figures, or the horizontal Sections of curve Surfaces, moving in Water, or any other Fluid.

Fig 14.

51. IF the Curve A M C moves in a Fluid, according to the Direction P R L perpendicular to its Axis A B; or, which is the ſame thing, if that Curve, being at reſt, receives the Impulſe of a Fluid in the Direction L R, and the Expreſſion of the mean Force of the Water, or Fluid, and the Line along which that Curve is diſpoſed to move, by the Preſſure of the Fluid, be required, we muſt firſt conſider the Curve as a Polygon of an infinite Number of ſmall Sides. Secondly, find (by the 30th Article) the Expreſſions of the lateral Forces, perpendicular and parallel to the Direction of the Fluid, upon one of the infinitely ſmall Sides of the Curve, conſidering the ſame as finite. Thirdly, thoſe Expreſſions will be infinitely ſmall Quantities, or the Fluxions of the lateral

Forces of the Fluid upon the whole Curve, ſo that their infinite Sums, or Fluxions, will be the Values of their lateral Forces, perpendicular and parallel to the Direction of the Fluid. Fourthly, according to the 40th, 42d, and 43d Articles, we take BE equal to the lateral perpendicular Force, and BF equal to the parallel one; then will the Diagonal BO of the Rectangle BFOE, denote the Tendency and Magnitude of the mean Force of the Impulſe of a Fluid againſt the whole Curve AMC.

52. Having called the Ordinates AP, x, PR, y, and drawn QM infinitely near PR: MR will be an infinitely ſmall Part of the Curve, and we ſhall have PQ or Rm, or pM $= \dot{x}$, Mm, or pR $= \dot{y}$; the right Line MR $= \sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}$. But now having found (by *Art.* 36.) by calling the Surface MR, a, the Sine of the Angle of Incidence MP, x, and its Complement PR, y, the lateral perpendicular Force RK $= \frac{xxy}{aa}$, and the lateral parallel Force RI $= \frac{x^3}{aa}$: and we have here $\dot{x}$ inſtead of x, $\dot{y}$ inſtead of y, and $\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}$, inſtead of a: ſo that only ſubſtituting $\dot{x}$, $\dot{y}$, and $\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}$, for x, y, and a, and we ſhall have the lateral Forces of the Impulſe upon the ſmall Side MR, *viz.* the perpendicular one RK $= \frac{\dot{x}\dot{x}\dot{y}}{\dot{x}\dot{x}+\dot{y}\dot{y}}$, and the parallel one KI, or RI $= \frac{\dot{x}^3}{\dot{x}\dot{x}+\dot{y}\dot{y}}$. Therefore BE will be the Fluent of $\frac{\dot{x}\dot{x}\dot{y}}{\dot{x}\dot{x}+\dot{y}\dot{y}}$, and BF the Flueut

Fig. 13, & 14.

of $\frac{\dot{x}^3}{\dot{x}\dot{x}+\dot{y}\dot{y}}$, and the Diagonal B O of the Rectangle E F, as has been before obſerved, will denote the mean Magnitude and Tendency of the moving Force, whereby the Fluid ſtrikes the Curve A M C. But if the Curve moves in the Fluid at reſt, the ſame Diagonal B O, will ſtill expreſs the Magnitude and Direction of the mean Reſiſtance, or, to move the Curve in the Direction P R L, in order to overcome the Reſiſtance of the Medium; it muſt be drawn or puſhed forwards in the Direction of the Diagonal B O, with a Force expreſſed by that ſame Diagonal.

53. But to obtain the Fluents or Values of the lateral Forces B E, B F, the Nature of the Curve A M C muſt be known, and then the Values of $\dot{x}$ and $\dot{y}$ in its Equation muſt be put into the general Expreſſions $BE =$ Fluent of $\frac{\dot{x}^2\dot{y}}{\dot{x}\dot{x}+\dot{y}\dot{y}}$, and BF $=$ Fluent of $\frac{\dot{x}^3}{\dot{x}\dot{x}+\dot{y}\dot{y}}$, and the Fluents afterwards found by the Rules laid down in the Doctrine of Fluents.

54. If the Curve A M C be a Circle, whoſe Radius is a, the Equation of the Curve will be $2aa - xx = yy$, and having thrown this into Fluxions, it will be $\dot{y} = \frac{a\dot{x} - x\dot{x}}{\sqrt{2aa - xx}}$, and $\dot{y}\dot{y} = \frac{aa\dot{x}\dot{x} - 2ax\dot{x}\dot{x} + xx\dot{x}\dot{x}}{2ax - xx}$. Then putting the Value of $\dot{y}\dot{y}$ into $\frac{\dot{x}^3}{\dot{x}\dot{x}+\dot{y}\dot{y}}$ the Fluent of the parallel lateral Expreſſion, and we have $\frac{\dot{x}^3}{\dot{x}\dot{x} + \frac{aa\dot{x}\dot{x} - 2ax\dot{x}\dot{x} + xx\dot{x}\dot{x}}{2ax - xx}}$

=

$= \frac{\dot{x}^3}{\frac{aa\dot{x}\dot{x}}{2ax - xx}} = \frac{2ax\dot{x} - xx\dot{x}}{aa}$, the Fluent of which, *viz.* $\frac{xx}{a} - \frac{x^3}{3aa}$, will be the Value of the lateral Impreſſion of the Fluid parallel to the Direction. alſo putting the Values of $\dot{y}$ and $\dot{y}\dot{y}$ into $\frac{\dot{x}\dot{x}\dot{y}}{\dot{x}\dot{x} + \dot{y}\dot{y}}$, the Fluxion of the perpendicular lateral Expreſſion, and we ſhall have $\dot{x}\dot{x}\dot{y} = \dot{x}\dot{x} \times \frac{a\dot{x} - x\dot{x}}{\sqrt{2ax - xx}} = \frac{\overline{a - x} \times \dot{x}^3}{\sqrt{2aa - xx}}$, and $\dot{x}\dot{x} + \dot{y}\dot{y} = \frac{aa\dot{x}\dot{x}}{2ax - xx}$. Therefore dividing $\frac{\overline{a - x} \times \dot{x}^3}{\sqrt{2ax - xx}}$ multiplied by $\overline{\frac{aa\dot{x}\dot{x}}{2ax - xx}}$ and we ſhall have $\frac{\dot{x}\dot{x}\dot{y}}{\dot{x}\dot{x} + \dot{y}\dot{y}} = \frac{\overline{a - x} \times \overline{2ax - xx} \times \dot{x}}{aa\sqrt{2ax - xx}} = \frac{\overline{a - x}}{aa} \times \sqrt{2ax - xx} \times \dot{x}$. Laſtly, by multiplying the Numerator and Denominator by $\sqrt{2ax - xx}$, and afterwards dividing it by $2ax - xx$, the Fluent of this Fluxion will be $\frac{\overline{2ax - xx}^{\frac{3}{2}}}{3aa} = \frac{\overline{2ax - xx} \times \sqrt{2ax - xx}}{3aa}$, being the Value of the lateral perpendicular Force. Wherefore we ſhall have BF : BE : : Fluent of $\frac{\dot{x}^3}{\dot{x}^2 + \dot{y}^2} = \frac{xx}{a} - \frac{x^3}{3aa}$: Fluent of BE $= \frac{\dot{x}\dot{x}\dot{y}}{\dot{x}\dot{x} + \dot{y}\dot{y}} = \frac{\overline{2ax - xx} \times \sqrt{2ax - xx}}{3aa}$.

55. If x be made $= a$, the Arch AMC will be a Quadrant, and BF will be $= \frac{2}{3}a$, and BE $= \frac{1}{3}a$, and the Diagonal BO $= a\sqrt{\frac{5}{9}}$ will determine the Magnitude and Tendency of the mean Reſiſtance of the Quadrant of a Circle.

56. If you deſire the Expreſſion of the Reſiſtance or Effort of the Fluid upon the Semicircle A C V, having made the Parallelogram E F for the Quadrant A C, and the Parallelogram e F for the Quadrant C V, it evidently appears (as in *Art.* 39, 41, and 42.) that the perpendicular lateral Forces do deſtroy each other, they being directly equal and contrary, and ſo the Semicircle will be moved in the Direction of the Fluid L F, with a Force expreſſed by B K the Double of B F, equal to $\frac{4}{3}a$. We may likewiſe conſider that each Quadrant of a Circle being moved with a Force expreſſed by B O, in the Directions B O, B o, the Diagonal B K of the Parallelogram B O K o, will, according to the Laws of compound Forces, be the Magnitude and Direction of the Force wherewith the Semi-Circle is determined to move by the Fluid.

57. An Arch of a Circle being given in Degrees, we may get numerical Expreſſions of the lateral Forces, by taking the Radius a equal to 100000, and ſubſtituting the verſed Sine of the given Arch for x. For Example, if the Arch A R be 30°, the verſed Sine x will be $= 19098$, and the Radius $a = 100000$; and putting theſe two Values of a and x, into the Expreſſions of the lateral Forces; and finiſhing the Computation, we ſhall have the parallel lateral Force $\frac{xx}{a} - \frac{x^3}{3aa} = 3415$, and the perpendicular lateral Force $\frac{\overline{2ax - xx} \times \sqrt{2ax - xx}}{3aa} = 6738$. We may obſerve

ſerve that ſince $\sqrt{2ax-xx}=y$ is the right Sine of the Arch, the perpendicular lateral Force is equal to the Cube of the right Sine divided by three times the Square of the Radius. And after this Manner we have computed the ſecond Table of lateral Forces for every 30 Minutes of the Arches of a Quadrant.

58. If the horizontal Sections of a Ship be made up of two circular Segments M A H, M *a* H; to determine the Situation of the Sail, according to her different Courſes and Lee-Ways; we firſt begin with the moſt ſimple Caſe, being that where the Lee-Way is nothing, or the Ships Courſe is in the Direction of the Keel. Now it appears plainly, that in this Caſe the Keel **B M** equally ſeparates the Efforts of the Reſiſtance of the Water made upon both the Sides M A, M *a*, of the Prow or Fore-part of the Ship; both theſe Sides being two circular Arches perfectly equal: ſo that the whole Reſiſtance, or Action of the Water upon theſe two Arches, being equal, the lateral parallel Forces, as well as the perpendicular ones, will be equal alſo. But (as in *Art.* 39.) the perpendicular Forces being equal and contrary, do deſtroy one another mutually; therefore, in this Caſe, the Reſiſtance the Ship meets with in paſſing thro' the Water, will be expreſſed by the Sum of the lateral parallel Forces, and the Line B G of the moving Force, will coincide with that of the Courſe B L, and the Keel B M; the Sail D C will be perpendicular to the Keel and Courſe, and the Ship ſail before the Wind. If the Parallels Fig. 17.

AL, *a* l, be drawn to the Courſe BL, the Space comprehended between theſe Parallels will ſhew the Breadth of the Water removed by the Motion of the Ship, or the naval Tract or Wake.

59. The Relation between the Length MH of the common Chord of the equal Segments MAH, M*a*H, and the Breadth A *a*, will eaſily ſhew the number of Degrees in the Arches MA, M*a*, againſt which we take (in the ſecond Table) both the lateral parallel Reſiſtances; and the Sum will be the Reſiſtance the Ship ſuffers by cutting the Water with her Stem. If, for Example, the Segments M A H, M *a* H, be both 30 Degrees, the Arches M A, M *a*, will be 15 Degrees: but the Line A *a*, which divides the Segments and common Chord into two equal Parts, being continued out both ways, paſſes through the Centres K and k of the Quadrants AMR, *a*Mr; ſo that the Courſe B L is perpendicular to the Radii A K, *a* K, and in the Table, over-againſt 15 Degrees you will find the lateral parallel Force to be $114\frac{762}{1000}$, the Double whereof $229\frac{524}{1000}$, will be the Expreſſion of the mean Reſiſtance that the Ship finds in cutting the Water, or the Value of the Side B F of the Rectangle of the lateral Forces. Therefore, in this Caſe, the other Side BE or FO is equal to nothing, becauſe the lateral parallel Forces do deſtroy one another by their Equality or Equilibrium; and ſo the Diagonal B O coincides with B F, and is equal to it.

Fig. 18. 60. If the Ship's Courſe be not parallel to the Keel, or the Ship has a Lee-Way meaſured by the Angle M B L, then to find the Line of the moving Force B G, and conſequently the Situation of the Sail D C, we muſt draw M N, H I, through M and H parallel to the Courſe B L, and having biſſected the Arches M N, H I, in the Points A, and *a*, through theſe we draw the Lines A K, *a* k, perpendicular to the Courſe B L; and both theſe Perpendiculars will paſs through the Centres K and k, of the Quadrants A M R, *a* M r, as being perpendicular to the Chords M N, H I. Now it is manifeſt, that the Action of the Reſiſtance of the Water is exerted only upon the Arches M A, M *a*; the Lines A l, *a* l, parallel to the Ship's Courſe, being Tangents at the Points A and *a*, ſo that all we have to do here, is firſt, to find the Value of the Arches M A, M *a*. Secondly, the Expreſſions of their lateral Reſiſtances, both perpendicular and parallel, in the ſecond Table. And thirdly, ſince, (as in *Art.* 42.) B F is equal to the Sum of the parallel lateral Forces, and B E equal to the Difference between the perpendicular lateral Force upon the Arch M A, and the ſame upon the Arch M *a*. For, as we have obſerved more than once already, that theſe Forces are directly oppoſite, and mutually deſtroy each other; the Diagonal B O will give the Magnitude and Tendency of the mean Reſiſtance. So that B G will be the Line of the moving Force, the perpendicular D C the Situation of the Sail, and the Space between the Parallels A l, *a* l, the Breadth of the Water of the Ship, or her Wake.

61. The Degrees in the Segment M A H, together with the Angle M B L, being known, to find from thence the Value of the Arches M A, M *a*, in the firſt place we may obſerve, that the Angles M H I, H M N, are equal to each other, and to the given Angle M B L, becauſe of the Parallels B L, H I, N M; but the Meaſures of theſe Angles are the half of the Arch M I, or N H: and conſequently, the Number of Degrees in each of theſe Arches is double that of the given Angle M B L. Therefore the Arches I A H, M *a* N, are both known, as alſo their Halves, I A, M *a*, and ſo the Arch M A. If, for Example, the Segments or Arches M A H, M *a* H, be 40 Degrees, and the Angle of the Lee-Way M B L 5 Degrees; the Arches M I, H N, will be 10 Degrees, and the Arches I A H, M *a* N, 30 Degrees, and ſo the Arch M A is 25 Degrees, and M *a* 15 Degrees. Then we muſt get the lateral Reſiſtances from the ſecond Table, and place them as follow:

	Parallel lateral Forces	Perpendicular lateral Forces.
An Arch of 25°	850 $\frac{367}{1000}$	2516
An Arch of 15°	114 $\frac{262}{1000}$	577 $\frac{667}{1000}$.

After which B F muſt be taken = 964 $\frac{639}{1000}$, *viz.* the Sum of the parallel lateral Forces, and B E = 1938 $\frac{333}{1000}$, the Difference of the perpendicular lateral ones. Laſtly, in the right-angled Triangle F B O, we ſay, as F B = 964 $\frac{639}{1000}$, is to F O =

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$1938\frac{333}{1000}$, ſo is the Radius, to the Tangent of the Angle F B O, equal to the Angle L B G, which will be found to be 63° 32′, and the Complement of this, or the Angle L B D of the Sail and the Courſe will be 26° 28′. Conſequently, the Angle M B D of the Keel and Sail will be 21° 28′.

62. It is evident, as the Angle M B L increaſes, the Arches M N, H I, do decreaſe, and when the ſame is meaſured by an Arch equal to one half the Segment M A H, the Lines H I, M N, are Tangents at the Points H, M. For the Angles M H I, H M N, can have only the Halves of the Arches M A H, or M a H, for their Meaſure, when the Lines H I, M N, are Tangents to thoſe Arches. Hence it is manifeſt, that then the Water acts by its Reſiſtance only upon the Side M A H; and conſequently, to obtain the Magnitude and Tendency of the mean Reſiſtance in this Caſe, we muſt aſſume B F equal to the parallel lateral Meaſure of the Force made by the Reſiſtance of the Water upon the Arch H M, and B E equal to the perpendicular lateral one: then will B O give the Tendency and Magnitude of the Mean Reſiſtance, and in the right-angled Triangle O F B, the Angle O B F will be found equal to the Angle L B G, whoſe Complement will be equal to the Angle of the Sail, and the Line of the Ship's Way, *&c.* If, for Example, the Arch M H be 40 Degrees, and the Angle M B L 20 Degrees, the parallel lateral Force upon that Arch will be 5046, and the perpendicular one $8852\frac{2}{3}$; and thence the Angle F B O will be found 60 Deg. 19 Min. conſequently

ly the Angle L B D of the Sail, and the Ship's Courſe is 29° 41′, and the Angle M B D, of the Keel and Sail 9° 41′.

Fig. 19. 63. If the Angle M B L be greater than one half of the Arch M H, or, which amounts to the ſame thing, if the Meaſure of that Angle be a greater Number of Degrees than one half the Segment M H, then, from the Centre K of the Segment, K A muſt be drawn perpendicular, and K R parallel to the Line B L of the Ship's Way, and the Quadrant A H M R, muſt be compleated. Then it is evident in this Caſe, as well as the laſt, that the Reſiſtance of the Water falls only upon the Side M H: ſo that to obtain the lateral Reſiſtances upon this Arch, we muſt take thoſe of the whole Arch A M from the Table, and from them thoſe of the Arch A H muſt be ſubtracted, and the Remainders will be the lateral Reſiſtances of the Arch H M; after which, as above, B F muſt be aſſumed equal to the parallel lateral Force, and B E equal to the perpendicular lateral one, and then we muſt work as before.

64. The Arch M H, and the Angle M B L, being given, to find the Arch A H, we muſt draw S H G parallel to B L, or perpendicular to the Radius K A: this done, it is evident the Arch S A, is equal to the Arch A G; but the Angle M H I, or M B L, has for its Meaſure one half of the Arch H M, plus one half of the Arch H S, or the Arch A H: and ſo to obtain the Degrees in the Arch A H, we muſt ſubtract the Degrees in half the Arch M H, from the Value of the

Angle M B L, and what remains will give the Value of the Arch A H. For Example, if the Segment M H be 30 Degrees, and the Angle of the Lee-Way M B L be 20 Degrees. Taking 15 from 20. and the Arch A H will be 5 Degrees; and conſequently, the Arch A M will be 35 Degrees: and in the ſecond Table I take the lateral Reſiſtances upon the Arches of 35 and 5 Degrees, which may be thus ſet down:

	Parallel lateral Reſiſtances,	Perpendicular ones.
An Arch of 35°	3073 $\frac{1}{3}$	6290 $\frac{68}{100}$.
An Arch of 5°	1 $\frac{45}{100}$	22.

Then, if the lateral Reſiſtances of the Arch of 5 Degrees be taken from thoſe of 35°, by rejecting the Fractions, we ſhall have the parallel lateral one upon the Arch H M 3073, for the Side B F, and the perpendicular one 6268 for the Side B E; from whence at length we find the Angle F B O = L B G to be 63 Deg. 53 Min. and its Complement L B D 26 Deg. 7 Min. the Angle M B G 83 Deg. 53 Min. and the Angle M B D of the Sail and the Keel 6 Deg. 7 Min. This is Mr. *Bernoulli*'s firſt Example, in his Treatiſe of the working of Ships, whoſe Calculation is infinitely longer.

65. If the Angle M B L of the Lee-Way ſo far encreaſes, as to cauſe the Point of the Prow M to fall beyond the Quadrant, or the Point R, this Caſe Fig. 20.

Caſe may be eaſily reſolved, if we conſider that the Angle M B L of the Lee-way is meaſured by one half of the Arch of the Segment H M, plus the Arch H A: therefore taking away one half of the Arch of the Segment from the Angle of the Lee-Way, and the Value of the Arch A H will be had, and alſo that of the Arches A H M, and M S. Then, from the lateral Reſiſtances upon the whole Quadrant A R, we take that of the Arch A H, to obtain thoſe of the Arch H R, and from the lateral ones upon the ſame Quadrant, or the Quadrant S R, we ſubtract thoſe of the Arch S M, to get thoſe of the Arch M R; and the Sum of the parallel lateral Forces upon the Arches R H, R M ſhall be the Value of B F, and the Difference between the perpendicular lateral ones of the ſame Arches, will be that of B E; for the reſt we proceed as in the Articles aforegoing.

Fig. 21. 66. Finally, if the Angle of the Lee-Way be a right Angle, in which Caſe the Sail will be parallel to the Keel, the Line of the moving Force will be, as in the firſt Caſe, the ſame as that of the Ship's Way, and the Side B E of the Parallelogram of the lateral Expreſſions will be equal to nothing; for in this Caſe B L divides the Segment M H equally at the Point I, which cauſes the perpendicular lateral Forces upon the Arches M I, H I, to deſtroy one another: and ſo B F or B O, equal to the Sum of the lateral parallel Forces upon the Arches M I, H I, ſhall be the Expreſſion of the Reſiſtance the Ship ſuffers in cutting the Water with her Side. But the Arch H I being known, the Arch H A is known too. Therefore to obtain the lateral parallel Reſiſtance upon the Arch H I; from the lateral one upon the whole Quadrant A I, we muſt take away that of the Arch A H, and we ſhall have that of the Arch H I, which being doubled, will give that of the Arch H M, or the Value of B F.

Of the ſix Caſes before-mentioned, the two firſt are the only ones that are uſeful in Practice; for, as we have obſerved before, (in *Art.* 23.) the nearer the Ship keeps up to the Wind, the more does the Angle of the Lee-Way increaſe; ſo that by trying to keep too near the Wind, ſhe will loſe by the Lee-Way, which, as we have obſerved, may cauſe her to be in danger, and fall off. The other Caſes, particularly the two laſt, do not take place, but when the Ship is drove with her Broadſide foremoſt.

68. By the firſt Caſe we can find the Expreſſion of the Reſiſtance the Ship finds in cutting the Water with her Stem, or Prow, and by the latter with her Side. So that knowing to what Segment of a Circle the horizontal Sections of a Ship may be referred; by thoſe Caſes may be found the Relation between the Reſiſtances made by cutting the Water with her Stem and Side. If the Segments be 20 Degrees, the Reſiſtance of the Side of the Ship will be 747 times greater than the Reſiſtance of her Stem: if the Angle of the Prow be 25 Degrees, the Reſiſtance of her Side will be 380 times greater than that of her Head; if 30 Degrees, 220; 35 Degrees, 138 times; 40 Degrees, Fig. 8.

92 times; 45 Degrees, 64 times; 50 Degrees, 46 times; 55 Degrees, 35 times; and 60 Degrees, 27 times. I have mentioned theſe Proportions of the Reſiſtances for Ships, which may be referred to Segments of every 5 Degrees, from 20 Degrees to 60 Degrees, becauſe theſe are the ſame for which I have made the Tables for the working of a Ship.

SECT. VI.

Of the Relation of the Velocities of a Ship, according to the different Courſes ſhe ſteers, and the ſeveral Situations of the Sails and Line of the Wind.

68. FROM the whole of what has been ſaid in the aforegoing Section, we may obſerve, that ſuppoſing a Ship moved in all her different Courſes with the ſame Velocity, the Diagonals B O will expreſs, in this Caſe, the ſeveral different mean Reſiſtances of the Water: but if the Velocities in different Courſes be different, the mean Reſiſtances (by *Art.* 5.) will be in a Ratio compounded of that of the Diagonals B O, and the duplicate Ratio of the Velocities. Let B O and b o, be the Diagonals, or the Expreſſions of the mean Reſiſtances of two different Courſes; then it is manifeſt, that if the Velocities be the ſame in every Courſe, the mean Reſiſtances will be as B O, to b o. But if V be the Velocity upon the firſt Courſe, and v that upon the ſecond; the mean Reſiſtances will be to one another as V V × B O, is to v v × b o. Fig. 12.

Now, if we ſuppoſe the Action of the Wind upon the Sail, or the moving Force to be the ſame upon different Courſes, that Force being always equal to the mean Reſiſtance, we ſhall have $VV \times BO = vv \times Bo$, and conſequently $VV : vv :: Bo : BO$; which ſhews that the Squares of the Velocities of a Ship, in her different Courſes, are in the reciprocal Ratio of the Diagonals of the Parallelogram of the lateral Reſiſtances. But $V : v :: \sqrt{Bo} : \sqrt{BO}$: wherefore the ſaid Reſiſtances are in the ſubduplicate reciprocal Ratio of thoſe Diagonals. But it is manifeſt that the greateſt Velocity of the Ship is when her Courſe coincides with the Keel, or the Lee-Way is nothing; and in this Caſe the Diagonal B O, is a Minimum.

69. Hence, if we take a Number, as 1000, for the Expreſſion of the greateſt Velocity, or the Velocity in the firſt Caſe; we may find the Velocities in all the other Directions by this Analogy, as the ſquare Root of the Diagonal B O, for a given Direction, is to the ſquare Root of the Diagonal of the ſtrait Direction, or that of the firſt Caſe; ſo is the Velocity 1000, to the Velocity of the given Direction. So that we muſt add the Logarithm of 1000 to the Logarithm of the ſquare Root of the leaſt Diagonal, and from that conſtant Sum, ſubtract the Logarithm of the ſquare Roots of every Diagonal, and the Remainders will be the Logarithms of the Velocities proportionable to the greateſt 1000. For Example, if you deſire the Velocity of a Ship, whoſe horizontal Sections may be referred to Segments of 30 Degrees; the

leaſt Diagonal, or leaſt Reſiſtance will be 230. The Logarithm of which is 236170, the half of which is 118085, for the Logarithm of the ſquare Root, to which adding the Logarithm of 1000, which is 300000, and the conſtant Sum 418085, will ſerve for all Caſes. If the Angle of the Lee-Way be 3 Degrees, from the Calculation in the right-angled Triangle B F O, we ſhall find the Logarithm of the Diagonal B O to be 286992, the half of which 143496, the Logarithm of the ſquare Root, being ſubtracted from 418085, and the Remainder 274589 will be the Logarithm of the Velocity, which will be found 557. If we take the greateſt Diagonal, or the Expreſſion of the Reſiſtance of the Ship with which ſhe cuts the Water with her Side, being 50608, whoſe Logarithm is 470422; and one half of it 235211, the Logarithm of the ſquare Root, being ſubtracted from 418085, the Remainder 182874, will be the Logarithm of 67, the Velocity with which the Ship would cut the Water with her Side, when drove by a Wind with the ſame Force that produces the Velocity 1000 to the ſame Ship, when cutting the Water with her Stem. And after this ſort it is that we have conſtructed our Tables of the Velocities of Ships for every different Situation of the Sail, Courſe, and Lee-Way.

70. The Proportions between the different Velocities of a Ship, which we have here ſhewn how to find, will be always the ſame, whether the Action or Force of the Wind againſt the Sails encreaſes or leſſens, provided the Encreaſe or De-

creaſe in either Caſe be equal. Now there are two principal Cauſes that produce the ſaid Increaſe or Decreaſe. Firſt, the different Angles of Incidence of the Wind upon the Sails; and ſecondly, the different Forces and Velocities of the Wind. But ſince the mean Reſiſtance of the Water is always equal to the Wind's Force upon the Sails, or the moving Force; the Diagonal BO, the Expreſſion of the mean Reſiſtance, muſt increaſe or leſſen by the ſame Quantity, whereby the moving Force, or that of the Wind upon the Sails increaſes or decreaſes. Therefore if whilſt a Ship is continuing upon the ſame Courſe, and the Sails always keep in the ſame Situation to the Keel, we ſuppoſe the Wind to change, or blow upon ſome other Point of the Compaſs, or, which is the ſame thing, the Sine of the Wind's Incidence upon the Sails to increaſe or diminiſh: then the mean Reſiſtance, or the Diagonal BO, will alter in Magnitude, but not Poſition. Let S be the firſt Sine of the Wind's Incidence upon the Sails, and s the ſecond. BO the firſt Diagonal, and Bo, the ſecond, and V the Ship's firſt Velocity, and v the ſecond. Now becauſe the Strokes are as the Squares of the Sines of Incidence, it will be as $BO : Bo :: SS : ss$, and $\sqrt{BO} : \sqrt{Bo} :: S : s$. But $V : v :: \sqrt{BO} : \sqrt{Bo}$. Wherefore $V : v :: S : s$. And ſo we ſee that the Velocities of the Ship in this Caſe are directly as the Sines of the Angles of Incidence of the Wind upon the Sails.

71. If at the Time a Ship keeps in the ſame Courſe, preſerving the ſame Situation of the Sails,

as well with reſpect to the Keel as the Line of the Wind, the Wind blows ſtronger or weaker; and it be required to find the various reſpective Velocities of the Wind upon the Sails, with ſome Exactneſs. In this Caſe the Velocities of the Ship are to one another in the ſame Ratio as the Velocities of the Wind: for (by *Art.* 2.) the Efforts of the Wind upon the Sails, being as the Squares of the Velocities, and the mean Reſiſtances of the Water being always equal to the Efforts of the Wind, or the moving Force; the Diagonal B O, keeping the ſame Situation, will encreaſe or diminiſh in the Ratio of the Squares of the Velocities of the Wind. Let X, be a firſt Velocity of the Wind, and x a ſecond: B O, the firſt Diagonal, and B o the ſecond. Then it will be as $XX : xx :: BO : Bo$, and $X : x :: \sqrt{BO} : \sqrt{Bo}$. Wherefore $V : v :: X : x$. Which ſhews that in this Caſe the Velocities of the Ship are in the ſame Ratio with the Velocities of the Wind.

72. From the three aforegoing Principles of the Velocities of a Ship, it follows, that when ſhe ſhapes her Courſe under different Angles of the Sails and Keel, and under different Angles of Incidence of the Wind upon the Sails; thoſe Velocities will be to one another in a Ratio compounded of the ſubduplicate reciprocal Ratio of the mean Reſiſtances, or the Diagonals BO, and of the ſingle Ratio of the Sines of the Angles of Incidence.

73. But if the Ship ſhapes her Courſe under different Angles of the Keel and Sail, with different Velocities of the Wind, whilſt the Angle of Inci-

dence of the Wind upon the Sails remains the ſame; the different Velocities of the Ship, will be to one another in a Ratio compounded of the ſubduplicate reciprocal Ratio of the Diagonals, and the ſingle Ratio of the Velocities.

74. The Angle of the Sails and Keel remaining the ſame, if the Sine of the Wind's Incidence upon the Sails varies; the Velocities of the Ship will be to one another in a Ratio compounded of the ſingle Ratio of the Sines of Incidence, and of the ſingle Ratio of the Velocities of the Wind.

75. Laſtly, if a Ship ſhapes her Courſe under various Angles of the Sails and Keel, various Angles of Incidence of the Wind upon the Sails, and various Velocities of the Wind: the Velocities of the Veſſel ſhall be to one another in a Ratio compounded of the ſubduplicate reciprocal Ratio of the Diagonals, of the ſingle Ratio of the Sines of Incidence, and the ſingle Ratio of the Velocities of the Wind. The Principles and Rules concerning the different Velocities of Ships here laid down are founded upon the Suppoſition that the Swiftneſs of the Wind is infinitely greater than that of the Ship, or that the Way ſhe falls to Lee-ward has no Compariſon to the Rapidity of the Wind. Whereas to obtain an accurate Determination of the Velocities, regard ſhould be had to the Way ſhe falls off from the Wind, or goes to Leeward, and the Winds reſpective Velocity againſt the Sails only taken, that is, the Exceſs of the Velocity of the Wind above that wherewith the Ship falls to Lee-ward. Now the Ship falls from the

Wind, when, as we have already obſerved, the Angle of the Ship's Courſe, and the Line of the Wind is obtuſe. But if this Conſideration were to be taken in, the Calculation would thereby become much more complex, and at the ſame Time no ſenſible Degree of Exactneſs for the Practice would be obtained therefrom. For firſt, it eaſily appears (by *Fig.* 3.) that the Velocity of the Ship, is to the Way ſhe falls to Leeward, as the Radius, is to the Sine Complement of the Angle L B A of her Courſe, and that of the Wind's; and conſequently, that Angle is ſo much the leſs, as the Angle L B A approaches to a right Angle, the Caſe wherein our Determinations are perfectly exact. Secondly, to obtain the reſpective Velocities of the Wind upon the Sails of a Ship which runs from the Wind, or falls to Lee-ward upon two different Courſes, we need only take Notice of the Difference of the Velocity upon the ſecond Courſe. But it is manifeſt the ſaid Difference muſt be very ſmall with regard to the abſolute Velocity of the Wind. Finally, it is very probable, the Chevalier *Renaud*, and Mr. *Huygens*, took no Notice of this Conſideration, as thinking the ſame to cauſe no ſenſible Difference in the Practice. Nor has Mr. *Bernoulli* done any thing about it.

76. Since a Ship often carries more or leſs Sail, to increaſe or diminiſh her Way; it being very plain the more Sails ſhe bears, the more is her Way increaſed; and contrariwiſe, the more one reefes the Sails, or, which is the ſame thing, the fewer Sails ſhe carries, the more is her Way leſſened,

ſened. It is therefore neceſſary to determine according to what Ratio her Way increaſes or decreaſes, when the different Magnitudes of the Sails are known, and this is eaſy to be done. For if S be the Surfaces of the Sails ſpread at one Time, and s the Surfaces of the Sails at another, the Velocities of the Wind, and the Angles of the Wind's Incidence upon the Sails, being ſuppoſed the ſame in both Caſes; it is manifeſt that the Forces of the Action of the Wind upon the Sails, will be proportional to the Surfaces of the Sails; ſo that if F expreſſes the Force of the Wind upon the Sails in the firſt Caſe, and f that in the ſecond, it will be $F : f :: S : s$. But the Reſiſtances or the Force of the Action of the Water upon the Prow of the Ship being always equal to the Force of the Wind upon the Sails, and ſince the Forces of the Water, are always as the Squares of the Velocities of the ſame Ship: if the Ship's Velocity in the firſt Caſe be called V, and in the ſecond v, we ſhall have $F : f :: VV : vv :: S : s$. Wherefore $V : v :: \sqrt{S} : \sqrt{s}$, which ſhews that the Velocities (or Ways Run) of a Ship, are in the ſubduplicate Ratio of the Surfaces of the Sails, or as the ſquare Roots of thoſe Surfaces.

77. Wherefore if the Surfaces of the Sails, and the reſpective Velocities of the Wind upon the ſame Sails be different, the Ship's Velocities, or Diſtances run, will be in a Ratio compounded of the ſubduplicate Ratio of the Surfaces of the Sails, and the ſingle Ratio of the reſpective Velocities of the Wind.

78. For the ſame Reaſons, if the Surfaces of the Sails, the reſpective Velocities of the Wind, and the Angles of the Wind's Incidence upon the Sails be different; the Ways made by the Ship in this Caſe, will be in a Ratio compounded of the ſubduplicate Ratio of the ſaid Surfaces, the ſingle Ratio of the reſpective Velocities of the Wind, and the ſingle Ratio of the Sines of the Incidence of the Wind upon the Sails.

79. Finally, if beſides theſe three Irregularities, the Angles of the Sails and Keel be alſo different, the Ship's Diſtances run, will here, be in a Ratio compounded of the ſubduplicate Ratio of the Surfaces of the Sails, the ſingle Ratio of the reſpective Velocities of the Wind, the ſingle Ratio of the Sines of the Wind's Incidence upon the Sails, and the ſubduplicate reciprocal Ratio of the Diagonals B O. All this is evident from what has been ſaid in the aforegoing Articles. It likewiſe appears that all we have ſaid here of the Velocities of the ſame Ship, is applicable to the Ratios between the Ways run of two or more Ships.

80. We have hitherto conſidered the Surfaces of Sails, as Planes, although at the ſame time every Sail filled by the Wind, is bent into a Curve, which Curve muſt needs produce ſome Alteration in the Direction of the Line of the moving Force; but, as is obſerved in the Preface, we have a very eaſy Way of reducing the Alteration cauſed by the Curvity of Sails, to an Alteration of plane Sails only. This Method, found out by Mr. *Bernoulli*, is very fortunate, by reaſon of its Simplicity. We

ſhall only lay down the Reſult thereof, referring thoſe who have a Mind to ſee its Demonſtration, to the Fifteenth Chapter of his Book.

Fig. 28. Let C D be a Yard, or the Plane of a flat Sail; its Perpendicular B G, will be the Line of the moving Force. But when the Wind comes to ſtrike this Sail in any Direction, as A B, and the Sail being flexible, it will become a Curve C G D. Now to find the Alteration that Curve cauſes in the Direction of the moving Force, we muſt, according to the Demonſtration of Mr. *Bernoulli*, draw mechanically, either by Gueſs, or with two Rulers, or two Threads, the Tangents C F or D F, to the Curve C G D, cutting one another in the Point F; and then the Line B F will be that of the moving Force. So that the Curvity of the Sail will change the Direction of the moving Force by the Quantity of the Angle F B G; and the Perpendicular C B D to B F, will be the Poſition of a plane Sail, producing the ſame Effect with that of the curved one C G D.

81. Becauſe in the Uſe of our Tables the Angle made by a flat Sail, or the Yard C D, with the Keel, is ſuppoſed to be known in Degrees; there ought to be for Practice ſome ſimple eaſy Method to find out the Angle that the Sail, as C B D, makes with the Keel, which muſt be looked upon as the Angle that the true Sail makes with the Keel, or that which we ought to find.

And amongſt ſeveral Ways I have thought upon for this Purpoſe, the moſt ſimple for Practice, in my Opinion, is this which follows. Take the

Quantities of the Angles F C D, F D C, with a Bevel, or otherwiſe, and make this Proportion. As the Sum of twice the Sine of the Angle F C D, and the Sine of the Angle C F D, is to their Difference; ſo is the Tangent of one half of the Complement of the Angle F D C to a Semicircle, to the Tangent of an Angle, which being added to one half of the ſame Complement to a Semicircle, will give the Angle D B F, whoſe Complement will be the Angle F B G, or D B d.

In order to demonſtrate this Rule, I call C D, $2a$, the Sine of the Angle C F D, S, and the Sine of the Angle F C D, s. This being ſuppoſed, we ſhall have by Trigonometry, $S : 2a :: s : DF = \frac{2as}{S}$, and in the Triangle B D F, the Sides B D, a, and D F, $\frac{2as}{S}$, will be known, together with the included Angle B D F: and therefore we form this Analogy, as the Sum of the two Sides $a + \frac{2as}{S}$, is to their Difference $\frac{2as}{S} - a$, ſo is the Tangent of one half the Sum of the unknown Angles, or of the Complement of the Angle B D F to a Semicircle, which I call t, to the Tangent of one half the Difference of thoſe unknown Angles. Which being added to one half the Sum of the unknown Angles; will give the Angle D B F, which was to be found. Therefore we muſt ſay, as $a + \frac{2as}{S} : \frac{2as}{S} - a :: t$ or $2s + S : 2s - S :: t : \frac{2st - St}{2s + S}$ = the Tangent of one half of the Difference. Which was to be demonſtrated.

Ex-

For Example.

If the Angle FDC be 60 Degrees, and the Angle FCD 50 Degrees; then will CFD be 70 Degrees, and we ſhall have $s = 76604$, $2s = 153208$, $S = 93969$. Therefore $2s - S = 59239$, and $2s + S = 212447$. We ſhall alſo find the Tangent $t = 173205$; and working the Proportion, it will be $\frac{2st - St}{2s + S} = 48296$, being the Tangent of 25 Degrees 46 Minutes, which being added to 60 Degrees, will give the Angle DBF 85 Deg. 46 Min. So that the Angle DBd will be in this Caſe 4 Degrees 14 Minutes.

SECT VII.

Of the Theory of the Rudder.

82. A Ship is navigated chiefly by two Motions; one cauſes her to deſcribe and ſhape the Courſe ſhe is to ſteer, and the other turns her about, and keeps her upon that Courſe. The former is cauſed by the Action of the Wind upon the Sails, and the latter by that of the Water upon the Rudder. The latter being almoſt as neceſſary as the former. If a Ship had no Rudder, or the Rudder could not govern her, how could ſhe readily turn about and ſteer her Courſe, and alter it according to Neceſſity? She would every Moment be then expoſed to great Danger, if ſhe could not tack about to avoid Sands and Shoals, and eſcape the Enemy, eſpecially in a Sea-Fight? It is a great Fault when a Ship is not governed by, or does not ſufficiently feel her Rudder, which to avoid, Shipwrights ought to take the greateſt Care. As the Bigneſs of the Rudder is but ſmall in compariſon to that of its Ship, the Effect thereof has

always appeared wonderful, even to the ancient Philoſophers themſelves, many of whom have in vain ſought to explain the Mechaniſm thereof, for want of knowing the Laws of the Impulſe of Fluids.

83. The Force of the Rudder depends upon two principal Cauſes. The firſt is, the Force of the Man ſuddenly puſhing the Helm, to turn the Ship to the Right or Left, or (as they ſay) Starboard or Larboard. And the ſecond, the Force of the Action of the Water, meeting the Rudder with a Velocity equal to that of the Ship's Way; this ſecond Cauſe being much more conſiderable than the firſt. And although the Chevalier *Renaud*, Mr. *Huygens*, and *Bernoulli*, ſeem to have looked upon the firſt as nothing, in compariſon of the ſecond, yet we ſhall explain them both.

Fig. 22. 84. In order to account eaſily for the firſt Cauſe, let us ſuppoſe the Ship HM, at reſt as in a Calm, and the Rudder HN to be firſt parallel with, or having the ſame Direction as the Keel, and afterwards a Man at the Helm puſhes the Tiller from HQ to Hq, and conſequently the Rudder to Hn, after the Reſiſtance of the Water is overcome. Now that Reſiſtance which the Force of the Man applied to the End of the Tiller Q, endeavours to puſh the Poop of the Ship into the Direction HK, and conſequently to turn her about. For let us for a Moment ſuppoſe the Reſiſtance of the Water to be infinite, ſo as that the Point R being the Middle of the Rudder, may be looked upon as fixed: then the Man's Force applied to

Q, will have the whole Length R Q, for the Arm of a Lever, and by puſhing the End of the Tiller from Q to q, will conſequently puſh the Point H towards the Point K. But during the Time that the Man does thus puſh the End of the Tiller from Q to q, he likewiſe with his Feet puſhes the Poop, or the Point H, with an equal Force the contrary Way, or towards the Point I: ſo that you ſee there are two Forces equal, and directly oppoſite, applied the one at Q and the other at H. But the firſt acts by the Arm of the Lever R Q, and the ſecond by the Arm R H of the Lever, and ſo the former will overcome the ſecond in the Ratio of R Q to R H. Conſequently the Point H, or the Poop of the Ship, will be puſhed towards the Point K: which muſt neceſſarily cauſe the Direction of the Keel H M to alter.

85. Now, I ſay, the Reſiſtance or Force of the Water re-united at the Middle R of the Rudder, not being infinite, the Ship's Poop will be always puſhed in the Direction R G, or r g, perpendicular to the Rudder, with a Force equal to that of the Reſiſtance of the Water againſt the Rudder. For let f be equal to the ſaid Force of the Water at the Point R, $HQ = m$, and $HR = n$. Then ſince the Rudder may turn freely about the Point H, it is evident the Man's Force applied at Q, is always proportional to the Force f, or if we ſay, $m : n :: f : \frac{nf}{m}$, this fourth Term $\frac{nf}{m}$ is always equal to the Man's Force employed in puſhing the End Q of the Tiller; for if the Man ſhould

puſh ſtronger, he would turn the Rudder O about ſwifeter, and conſequently increaſe the Force f: wherefore the Tiller QHR, or qHr, is puſhed at the ſame Time in the perpendicular Direction RG, or rg, with two Forces, f, and $\frac{nf}{m}$; the one at R, or r, the other at Q, or q. But whilſt the Man puſhes the End Q of the Tiller, he likewiſe puſhes the Poop with his Feet, and conſequently the Point H, with a Force directly oppoſite and equal. That is, at the ſame Time the Tiller QHR is puſhed with a Force equal to $f + \frac{nf}{m}$, it is drawn away to the Point H, with a Force equal to $\frac{nf}{m}$, ſo that there only remains the Force f, by which the Poop is puſhed in a Direction perpendicular to the Rudder. It evidently appears, that the Ship will turn ſo much the ſwifter about as the Force applied at Q is greater. For the Force f will increaſe according as the Tiller is turned more ſwiftly about.

This firſt Cauſe takes place, whether the Ship makes any Way or not; but the ſecond does not, unleſs the Ship makes ſome Way.

86. In order to explain eaſily the Mechaniſm of the ſecond Cauſe, in the firſt Place it may be obſerved, that when the Rudder lies parallel to the Ship's Courſe, or makes an Angle equal to two right Angles with the Line the Ship moves along, in this Caſe there is no Impreſſion at all received by the Rudder from the Water; and conſequently ſo long as the Ship remains in this Situation, ſhe

purſues the ſame Courſe. But when the Courſe B L, or B l, parallel to it, makes the Angle l R H with the Rudder, the Surface H N of the Rudder, receives the Stroke of the Water with a Velocity equal to that of the Ship's Way, under the Angle of Incidence l R H. But having (according to *Art.* 36.) reſolved the whole Force of the Water's Impreſſion, repreſented by R G, into the two lateral ones R K, K G, perpendicular and parallel to the Direction R l, or B L of the Courſe, it then evidently appears that the perpendicular lateral Force exerts itſelf by puſhing the Poop of the Ship into the Direction R K, perpendicular to the Ship's Courſe, and making her turn at the ſame Time as the parallel lateral Force K G leſſens and retards her Way; as being directly contrary to it. From whence we may obſerve, that a Ship will turn about ſo much the ſwifter, as the lateral Force R K is greater. But it is manifeſt, the ſaid Force may increaſe for two Reaſons. Firſt, from the Velocity of the Ship, or, which is the ſame thing, the Velocity of the Water upon the Rudder. And ſecondly, from the Angle of Incidence l R H. But during the Time that we are turning the Ship, or altering her Courſe, her Velocity may be looked upon as equable; therefore to turn the Ship about in the ſhorteſt Time poſſible, the Sine of the Angle of Incidence l R H muſt be found, ſo as the greateſt perpendicular lateral Force R K, may reſult therefrom; and that Angle will give the moſt advantageous Situation of the Rudder.

Fig. 23.

87. Therefore to find the Sine of the Incidence affording the greateſt lateral Force R K, aſſume (as in *Art.* 36.) H R for the Radius, and then H P will be the Sine of the Incidence, make H R $= a$, H P $= x$, and R P will be $= \sqrt{aa-xx}$. Then (as in *Art.* 36.) the Value of the perpendicular lateral Force R K will be $= \frac{xx}{aa}\sqrt{aa-xx}$. Which muſt be a *Maximum.* Therefore making its Fluxion equal to 0, and we ſhall have $\frac{2x\dot{x}}{aa}\sqrt{aa-xx} - \frac{x^3\dot{x}}{aa\sqrt{aa-xx}} = 0$. From whence we obtain $2\sqrt{aa-xx} = \frac{xx}{\sqrt{aa-xx}}$, by multiplying all by aa, and dividing by $x\dot{x}$. And laſtly, if both Sides of this Equation be multiplied by $\sqrt{aa-xx}$; we ſhall have $2\times\overline{aa-xx} = xx$; and ſo $2aa = 3xx$, and $\frac{2}{3}aa = xx$. And therefore the Sine of the moſt advantageous Angle of Incidence of the Rudder and Courſe, will be $\sqrt{\frac{2}{3}aa}$; the correſpondent Arch of which is 54 Deg. 44 Min. This beſt Situation of the Rudder being the very ſame as that found out by the Chevalier *Reneaud*, Mr. *Huygens*, *Bernoulli*, and *Guinée*, about which we have ſpoke in *The Memoires of the Royal Academy*, *for the Year* 1727, *page* 59. But we muſt obſerve that though all theſe Gentlemen ſuppoſed the Ship to be exempt from, or really have no Lee-Way, yet we have had Regard thereto; becauſe our taking the Line wherein the Head of the Ship begins to turn, as perpendicular to the Line of the Ship's Way, makes the Queſtion come to the very ſame thing.

88. But if when the Lee-Way is conſidered, we always take the Line wherein the Head of the Ship begins to turn as perpendicular to the Keel; the whole Force R G muſt be reſolved into two lateral ones, perpendicular and parallel to the Direction of the Keel; but becauſe of the Lee-Way, the Situation of the Keel, or the Direction in which the whole Force is reſolved, being different from that of the Fluid, the Queſtion becomes the very ſame as that of *Art.* 27. that is, having drawn H S perpendicular to the Direction of the Water, H D perpendicular to the Keel, and having made H R (equal to the Radius) $= a$, the Sine of the Angle of Incidence H P $= x$, and H C the Coſine of the Angle S H D (equal to the Angle M B L or M H L) $= b$; we ſhall find. as in *Art.* 28. that the Sine of the Incidence for the greateſt lateral Force R K, perpendicular to the Keel, will be $x = \sqrt{\frac{1}{2}aa + \frac{1}{6}bb \pm \frac{1}{2}\sqrt{a^4 - \frac{10}{9}aabb + \frac{1}{9}b^4}}$. And it is plain, that as the Angle of the Lee-Way M H l, equal to the Angle S H D, diminiſhes the Sine H C $= b$, increaſes, and when that Angle becomes nothing, the Point C falls upon S, and b will be $= a$, and putting b for a in the Equation $x = \sqrt{\frac{1}{2}aa + \frac{1}{6}bb \pm \frac{1}{2}\sqrt{a^4 - \frac{10}{9}aabb + \frac{1}{9}b^4}}$; we ſhall get $x = \sqrt{\frac{2}{3}aa}$, for the Caſe wherein the Lee-Way is nothing. Laſtly, if the Planking of the Ship alters the Stream or Direction of the Wather againſt the Rudder; we muſt firſt take notice of that Direction, and find, as above, the greateſt lateral Force perpendicular to the Ship's Courſe.

SECT. VIII.

Of the Theory of the Oar.

89. ALthough the Mechaniſm or working of the Oar, appears to be very plain and ſimple, yet it deſerves ſome Explanation, to underſtand and determine, 1°, what is the Quantity of the Force puſhing, and cauſing the Boat or Galley to move on, ariſing from the Force of the Watermen when they work the Oars. And 2°, what is the moſt advantageous Length to be given to the Shank, or Part of the Oar from the Rowlock, or Point upon which the Oar turns, to the Middle of the Blade, or the Centre of the Reſiſtance of the Water.

90. And firſt of all, to determine the Force wherewith the Oars puſh, and endeavour to make the Boat or Galley move forward, ariſing from the Force of the Watermen, it is plain that we need only conſider one Oar QHR only, moved by a Force applied to its End Q, which may be conſidered as that of one Man only; the Point H is the *Rowlock* in which the Oar plays, ſo as to turn about

Fig. 24.

about freely; and the Point R, the middle of the Blade, or the Centre of the Reſiſtance of the Water: now, whilſt the Man exerts his Force to draw the End Q, in the Direction QS parallel to the Boat's Way, or Courſe BL, the Reſiſtance the Blade R ſuffers in cutting the Water, acts, by puſhing back the Middle R of the Blade, in the Direction RT parallel to QS; ſo that the ſaid Point H is to be taken as the Fulcrum of the two Levers HQ, HR: Therefore this Point will be drove forwards in the Direction HG parallel to the Courſe, with a Force equal to the Sum of the Forces of the Man at the Point Q, and that of the Reſiſtance of the Water at the Point R; but ſince at the ſame Time as the Waterman endeavours to draw the End Q of the Oar, he puſhes back the Boat with his Feet the contrary Way, by a Force equal to that wherewith he pulls the Oar; and conſequently the Point H being drawn in the Direction HI with a Force equal to that of the Waterman, the ſole Force remaining to carry on the Point H in the Direction HG, and make the Boat go forwards, is equal to that of the Water's Reſiſtance againſt the Oar. What is mentioned of one Oar, is equally applicable to any Number whatever that a Boat or Galley may have; as alſo the Force of one Rower only may be conceived as that of ſeveral applied to one ſingle Oar, coinciding in a mean, or middle Point Q. And from hence we may conclude, that the Quantity of the Force employed to move forwards a Galley without

out Sails, is always equal to the Sum of the Reſiſtances of the Water againſt all the Oars.

91. Now in order to compare the Force of the Rowers with the Reſiſtances of the Water againſt the Blades of all the Oars; and conſequently the Force wherewith the Galley moves forwards: we will firſt ſuppoſe the Centre or Middle R of the Blade to be a fixed Point, and conceive that whilſt the Rower pulls the Point Q in the Direction QS, he acts by the Arm of a Lever QR: and ſo his Effort will be to all the Effort moving the Point H in the Direction HG, as HR to QR, if at the ſame Time the Point H were not puſhed back the contrary Way. Let the Length QR be $= a$, $HR = x$, and conſequently $QH = a - x$. If f be taken equal to the Force of the Waterman upon his Oar; then becauſe of the Arms of the Lever, it will be $HR : QR :: x : a :: f : \frac{af}{x} =$ the Force wherewith the Point H is carried forwards in the Direction HG, by Means of the Power applied at Q. But we have obſerved, that the Waterman being, as it were, fixed in the Boat, his Re-action draws back the Point H in the contrary Direction HI, with a Force equal to that he exerts at the Point Q. Wherefore $\frac{af}{x} - f = \frac{af - xf}{x}$ will expreſs the Force cauſing the Boat or Galley to move forwards, which Force (as has been obſerved) is equal to the Action or Reſiſtance of the Water againſt the Blade R; this may be demonſtrated likewiſe by aſſuming the Point H

for a Fulcrum to the Levers HR, HQ, in order to get $x : a - x :: f : \frac{af - fx}{x}$ equal to the Reſiſtance applied at R, exactly the ſame. For Example, if the Oar be 18 Feet, the Diſtance HR from the Rowlock to the Centre of the Blade 10 Feet, and the Force of the Waterman 30. Then will $a = 18$, $x = 10$, and $f = 30$. From whence we get $\frac{af - fx}{x} = \frac{18 \times 30 - 30 \times 10}{10} = \frac{540 - 300}{10} = 24$, the Expreſſion of the Reſiſtance of the Water, or the Force cauſing the Boat to move forwards, by Means of the Power f.

92. Now to obtain the moſt advantageous Ratio of the Lengths QH, and HR; that is, to find the Point Q of the Oar, which the Waterman muſt pull to produce the greateſt Effort poſſible; the Point H being the Centre of Motion of the Man's Arms applied at Q, the Force f acts by the Arms of the Lever $QH = a - x$; ſo that the Momentum or Quantity of Motion of the Power applied at Q to draw the Point H, will be $af - fx$. This being granted, and the Point R, (as has been already obſerved) be ſuppoſed to be fixed, the Point H at every Stroke of the Oar, or if you will, every Inſtant, will deſcribe an Arch, having HR for its Radius. But ſince it is evident, that only ſome certain Number of Strokes of the Oar can be made in a given Time, the Quantity of the Way moved through by the Boat or Galley in a given Time, depends upon the Bigneſs of that Arch; ſo that for this Reaſon, the Galley will run ſo much the faſter as the Diſtance

from the Rowlock to the Centre of the Blade, or the Length H R, is greater. But by how much the Side H R (x) increaſes, ſo much does the Side H Q ($a-x$) decreaſe, and conſequently, the Momentum of the Waterman's Power decreaſes too. Therefore ſuch a Ratio between Q H, and H R muſt be found, as that the Product of the Momentum $af-fx$ into the Arch deſcribed by the Point H, or, which is the ſame, by its Radius H R (x) ſhall be the greateſt of any ſuch Product. But this is very eaſy to find: for ſince the Product of $af-fx$ by x, is $afx-fxx$, and its Fluxion is $af\dot{x}-2fx\dot{x}$; this, according to the Method of Maximums, muſt be equal to 0; and then we ſhall have $2x=a$, and R H, or $x=\frac{1}{2}a$. Which ſhews, when a Waterman uſes his greateſt Force in rowing, the Point of the Oar whereat he places his Hands, and the Centre R of the Blade, muſt be at equal Diſtances from the reſting Point H.

93. When ſeveral Rowers work at one Oar, their various Forces muſt be conceived as united at a middle Point Q, in the Diſtance A C, whereat they work, and Q R muſt be divided into two equal Parts, to obtain the Point H that is to be put into the Rowlock.

SECT.

SECT. IX.

Of the Uſe of the Tables: together with the Reſolutions of the chief Queſtions about the working of Ships.

94. THE firſt Table ſerves only for the Conſtruction of Tables for the working of Ships, whoſe horizontal Sections are taken for Polygons, as we have explained in the 44th Article.

The ſecond ſerves for the Conſtruction of Tables for the working of Ships, whoſe horizontal Sections are Segments of Circles, as we have likewiſe explained in the 57th Article. Theſe Tables have many more Uſes than what we here explain; to lay down which would be foreign to our Deſign.

The third Table contains the moſt advantageous Situations, both of the Sails and Keel, either when the Ship goes to Windward, or runs large, as we have explained in the 30th Article.

The 4th, 5th, 6th, 7th, 8th, 9th, 10th, 11th and 12th Tables, in the firſt Place contain the

Angles of the Keel and Ship's Courſe for every 30 Minutes, anſwerable to the Angles of the Sails and Ship's Courſe, computed according to the Methods laid down in the 59th, 61ſt, and 64th Articles. Theſe being the Tables deſigned by Mr. *Bernoulli*, and do contain the Relations of the Velocities, the greateſt of which is 100, computed by the 69th Article, and are of very great Uſe.

95. Laſtly, thoſe ſame Tables do contain the Relations of the Quantity of the Lee-Way, which are computed very eaſily. For if B L be the Length run by a Ship in a given Time, and B P be taken equal to B L, the right Line P L will be the Quantity of the Lee-Way. Or, if the Diſtance run B L, be taken for the Radius, P L will be the Chord of the Angle of the Lee-Way: ſo this Table only contains the Values of the Chords of the Angles of the Lee-Way, when the Radius is only 1000: it being ſufficient for Practice to multiply the Ship's Diſtance run by the Chord of the Angle of the Lee-Way found in the Table, and then cut off the three laſt Figures from the Product, and the Remainder is the Lee-Way. Fig. 12, & 18.

96. The 13th, 14th, 15th, 16th, 17th, 18th, 19th, 20th, and 21ſt Tables, are made for Practice; (it is neceſſary here to take notice, that we have not been ſtrictly exact, becauſe we have made uſe of the third Table, wherein Ships are ſuppoſed to have no Lee-Way) ſo that the Situations of the Sails are not what they ſhould be exactly. But one or two Degrees in the Poſition of the Sails, will not be ſenſible in Practice.

In order to conſtruct theſe Tables, we have firſt aſſumed the Angles of the Courſe of the Wind and the Keel for every three Degrees; which is ſufficient for Practice. Secondly, with theſe Angles, in the third Table, we have taken the moſt advantageous Angles of the Sails and Keel, both when the Ship obtains the Wind, or runs to Windward and ſails with the Wind, or at large, as alſo the moſt advantageous Angles of the Sails, and Line of the Wind. Thirdly, with the Angles of the Sails and Keel, we have taken, in the 4th, 5th, and 6th, *&c.* Tables, the Lee-Way Angles, which we have added to the Angles of the Courſe of the Wind and Keel, thereby to obtain thoſe of the Courſe of the Wind, and that of the Ship. Fourthly, we have taken the Expreſſions or Relations of the Velocities in the ſame 4th, 5th, and 6th Tables, anſwerable to the Angles of the Sails and Keel. Fifthly, with the moſt advantageous Angle of the Rudder, in the Caſe where the Lee-Way is nothing, which is 54° 44′, and the Angles of the Lee-Way, we have ſet down the moſt advantageous Angles of the Rudder, to turn about the Veſſel, with the Wind before and aft.

97. In every Voyage the Angle of the Wind's Courſe, and that whereon the Ship is to run, is always known or given; for otherwiſe, it muſt be firſt determined, from the Principles of Piloting. But it is evident by all the Laws of the working of Ships, that the Lee-Way is ſo much the ſmaller, as that Angle is more obtuſe, and ſo much the greater as the ſame is more acute.

But as it is very difficult to be certain to half a Degree of the Angle of the Wind's and Ship's Courſe, or of the Angle made by the Meridian of the Compaſs, and the Line of the Ship's Courſe, we look upon every Angle of the Lee-Way leſs than 30 Minutes, to be inſenſible, and not to be regarded; ſo that when that Angle is leſs than 30 Minutes, we ſay there is no Lee-Way.

The Lines of the Direction of the Keel, and thoſe of the Ship's Courſe, do differ from one another by the Quantity of the Lee-Way, and yet it is a general Cuſtom amongſt Pilots, and all thoſe who are employed in conducting Ships, to direct the Keel along the Line of the Courſe; which by ſo doing, they deviate from the true Courſe the Quantity of the Lee-Way. It is true, indeed, that not knowing the Quantity of the Lee-Way, they take the ſafeſt Method they can, which they correct afterwards by the Rules of piloting. And ſometimes it even happens, when the Keel is not kept in the Direction of the Courſe, or her Head pointing to the Courſe, that this Error is corrected by the Lee-Way; and then we ſay the Lee-Way makes good the Courſe.

98. When the Angle of the Lee-Way is known, the Ship may be always kept to her Courſe, by making the Angle of the Keel and Courſe equal to the Angle of the Lee-Way. But to do ſo, we muſt have Tables exhibiting the Lee-Ways anſwerable to the ſeveral Courſes. I believe myſelf to be the firſt who has given ſuch Tables, and dare ſay their Uſe will more and more demonſtrate the Ne-

Neceſſity for them. But as it is not eaſy to ſet aſide eſtabliſhed, though faulty Cuſtoms, eſpecially in Navigation. I foreſee that theſe Tables will require ſome Time gradually to eſtabliſh a Reputation.

99. Seamen have found by Experience, that there is a certain Point unto which a Ship upon a Wind may go the neareſt, although they could never determine it: but we perceive with Pleaſure, that our Tables do determine what that Point is, and even the Computation has ſhewn us how to find it; and it is in fact perceived that they need not be carried further on.

We have therefore continued our Tables ſo far, as to ſhew the ſmalleſt Angle that the Line of the Keel can make with the Line of the Wind, beyond which, as has been obſerved in the 24th Article, if you ſhould ſtill keep cloſer up to the Wind, by leſſening that Angle; the Angle of the Line of the Wind, and that of the Courſe, will be increaſed in too great a Ratio by the Lee-Way.

100. As the greateſt Lee-Way happens neceſſarily in that very Point wherein the Ship keeps cloſeſt up to the Wind, and likewiſe the augmenting or leſſening the Angles of the Sails and Keel, have not been, according to any certain Rules, hitherto laid down; the beſt Seamen have looked upon the greateſt Lee-Way as very uncertain; ſome making it from 15 to 16 Deg. others 22 Deg. and others 30 Deg. though it be true that two Ships rarely have their greateſt Lee-Ways equal, yet by our Tables we may remove that Difficulty, and find

the greateſt Lee-Way of a Ship, as near as is neceſſary for Practice.

101. But what moſtly cauſes Veſſels to make Lee-Way, and hinder them from keeping as cloſe upon a Wind as poſſible, is, that moſt Seamen, when they lie cloſe-hawl'd, make the Angle of the Sails and Keel too ſmall. Indeed, it cannot be denied, but by this Means the Sails gain the moſt Force poſſible of the Wind; but then it is eaſily ſeen, that almoſt all that Force is exerted in puſhing the Ship ſide-ways, and making her fall off from the Wind more, than if ſhe were not kept up ſo cloſe to the Wind. An Example of which we have in Captain *Cowley*'s *Voyage round the World*, who finding himſelf under a Neceſſity of keeping as cloſe to the Wind as poſſible, ſays, that *upon the Noon of the 29th Day of* May, *in the Year* 1686, *the Wind ſhifting Weſt-South-Weſt, we ſteer'd North-Weſt with the Top-gallant Sails ſet, but the Ship could never be kept ſo near the Wind, and run only between the North-Weſt and Weſt-North-Weſt, &c.* From whence it appears, that the Keel made an Angle with the Line of the Wind of 67 Deg. 30 Min. and the Lee-Way was 22 Deg. 30 Min. Now I find the Reaſon of the Captain's having ſo great a Lee-Way was, his making the Angle of the Sails and Keel but 14 Degrees, when he might nevertheleſs have made the ſame at leaſt 24 Deg. and he would have kept cloſer to the Wind by one whole Point, or nearly thereabouts.

In order to give a more compleat Account of the Uſe of my Tables, I ſhall lay down ſome Examples, in form of Problems upon the working of Ships, and that chiefly upon Veſſels whoſe Prow makes a Curve-line Angle of 30 Degrees, or whoſe horizontal Sections of the Hull are nearly Segments of 30 Degrees, joined by a common Chord, repreſenting the Keel, which is the Suppoſition of Mr. *Bernoulli*; it being eaſy to perceive that the Prow or Fore-Part of a Ship has nearly ſuch a Shape; and the Shape of the Poop, be it what it will, is of no Conſequence.

102. Since in the Practice of Navigation there is no promiſing one's ſelf an Exactneſs of one Degree, more or leſs, and even oftentimes ſeveral Degrees, by reaſon, as we have obſerved, of Currents, and ſometimes for want of the true Knowledge of the Variation of the Compaſs; if the Angle of the Ship's Courſe and Line of the Wind be given in Points, we reduce them into Degrees by allowing 11 Deg. 15 Min. for every Point; and with theſe Degrees we enter the Tables under the Column of Angles of the Line of the Wind and the ſeveral Courſes, and if theſe Angles be not exactly to be found, we take them as near as may be.

PROB. I.

A Ship's Courſe being given, together with that of the Wind: to find the Angle made by the Sails and Keel, and alſo the Lee-Way.

103. This Problem is quite reſolved by the Tables; for if the Difference between the Ship's Courſe and that of the Wind, be expreſſed in Points, we reduce them into Degrees, by allowing 11 Degrees 15 Min. for every Point; and theſe will be the Angle of the Ship's and Wind's Courſe, which being found in the Table, or that which is the neareſt to it, and over-againſt the ſame in the other Columns, will be found the Angles of the Sails and Keel, the Lee-Way, *&c.*

EXAMPLE I.

104. The Wind being at Eaſt, I want to ſail Weſt-North-Weſt. Here the Diſtance between the Ship's Courſe and that of the Wind is 14 Points, or 157 Deg. 30 Min. And ſince in our Examples we take ſuch Ships whoſe Prows make an Angle of 30 Degrees, we look for 157 Deg. 30 Min. in the Column of the Angles of the Ship's and Wind's Courſe, in the 15th Table; and as it cannot be exactly found, we take 158 Degrees, the neareſt to it, and over-againſt the ſame will be found 75 for the Angle of the Sails and Keel, for we may reject the Minutes; the Angle of the Sails and the

Courſe of the Wind 83 Degrees. And ſince the Lee-Way is leſs than 30 Minutes, we eſteem it as nothing: ſo that the Angle of the Wind's Courſe and the Keel is alſo 158 Degrees. And in this Caſe the Direction of the Keel ought to coincide with that of the Ship's Courſe.

EXAMPLE II.

105. The Wind being North-Weſt, I want to ſail Weſt-South-Weſt.

The Difference between the Wind's Courſe and the Ship's, is 6 Points, or 67 Deg. 30 Min. which cannot be exactly found in the Table of the Angles of the Wind's and Ship's Courſe: in this Caſe, I take 66 Deg. 30 Min. being the neareſt to it; over-againſt which we find the Angle of the Sails and Keel to be 23 Deg. 43 Min. or almoſt 24 Degrees. And the Angle of the Sails and Keel 40 Deg. the Lee-Way 2 Degrees 30 Minutes, and the Angle of the Wind's Courſe, and that of the Keel 64 Degrees. From whence you may ſee in this Caſe, that for the Lee-Way to make good the Ship's Courſe, the Head of the Ship muſt be carried 2 Deg. 30 Min. to the Weſtward of the Ship's Courſe; and then ſhe will run as cloſe as neceſſary to make good her Courſe.

EXAMPLE III.

106. The Wind being North-North-Eaſt, I want to ſail South-Eaſt 7 Deg. 30 Min. South, which is 10 Points and 7 Deg. 30 Min. or 120 Degrees, being the Angle of the Ship's and Wind's Courſe, which I find exactly in the Table, and over-againſt

gainſt it 51 Degrees for the Angle of the Sails and Keel, 68 Degrees for the Angle of the Wind's Incidence upon the Sails; 1 Degree of Lee-Way, and the Angle of the Courſe of the Wind and Keel 119 Degrees. And therefore, ſince the Lee-Way makes good the Ship's Courſe, her Head muſt point a Degree farther Eaſtwards than is the Bearing of the Place the Ship is to ſail to, and ſhe will run large.

PROB. II.

To determine the Ratios of the Velocities of the Veſſels of the three Examples aforegoing, when the Action or Force of the Wind againſt the Sails in all the three Caſes is the ſame.

107. In the Column of Velocities, over-againſt the Angle of the Wind's and Ship's Courſe, take the Expreſſions of the Velocities for each Example, and that of the firſt will be found to be 972, that of the ſecond 603, and that of the third 902. Theſe three Velocities therefore are to one another, as the three Numbers 972, 603, and 902; one of which being known in Leagues *per* Hour, or *per* Day, the two others will be known too. If the firſt, for Example, be 3 Leagues in an Hour, or 72 in a Day, to get the Velocity of the ſecond Example, we ſay, If 972 gives 72 Leagues, 603 will give $44\frac{2}{3}$ Leagues; and of the third we ſay, if 972 gives 72 Leagues, 902 will give 66 Leagues.

Leagues, or thereabouts: therefore we ſee that the ſame Ship (of Example I.) running 72 Leagues a Day, will run along the Courſe of the ſecond Example $44\frac{2}{3}$ Leagues, and along that of the third 66 Leagues and $\frac{7}{9}$. So that having but once taken an exact Account of the Ship's Run or Way upon any given Courſe, by this Problem we can find her Way upon any other Courſe; provided the Action of the Wind upon the Sails be the ſame. But ſince when the Courſe changes, the Angle of the Incidence of the Wind is neceſſarily changed too; the Wind's Action againſt the Sails cannot be the ſame upon various Courſes.

PROB. III.

To determine the Ratios between the Velocities of a Ship on her different Courſes, and the different Angles of Incidence of the Wind againſt the Sails.

108. We ſhall aſſume the three Examples aforegoing, which determine the Velocities of a Veſſel by taking in the Ratios between the different Forces of the Wind upon the Sails, and by the Angle of the Incidence of the Line of the Wind upon the Sails, which is very eaſy to do; for, by the 70th Article, if the Angle of Incidence of the Line of the Wind upon the Sails changes, whilſt that of the Sails and Keel remains the ſame, as well as the Velocity of the Wind; then will the Ship's Runs

or Ways be in the ſingle Ratio of the Sines of the Wind's Incidence upon the Sails. So that bringing into the Ratios of the Velocities of the three Examples aforegoing, that of the Sine of the Incidence of the Wind upon the Sails; and the required Velocities will be had. The Velocity of the firſt Example is 972, that of the ſecond 603, and that of the third 902. And in the Table you will find the Angle of the Sails and Line of the Wind for the firſt Example, 82 Deg. 35 Min. or 83 Degrees; for the ſecond 40 Degrees: and for the third 68 Degrees. We have then theſe three Angles, 83, 40, and 68, whoſe Sines are 99254, 64279, and 92718. Multiplying the firſt by 972, the ſecond by 603, and the third by 902, the three Products 96474888, 38760237, and 83631636, will be had, expreſſing the Velocities in the three Caſes. Laſtly, if, as in the foregoing Problem, we knew the Ship's Way in one of the three Caſes, her Way in the two others may be found by the Rule of Proportion. If, for Example, in the firſt Caſe, the Ship's Way is 72 Leagues in 24 Hours, to obtain her Way in the ſecond, we ſay, if 96474888 gives 72, what will 38760237 give? When the Rule is finiſhed, it will be found 28 Leagues and $\frac{9}{10}$; and for the third, if 96474888 give 72, what will 83631636 give? the Anſwer will be 62 $\frac{4}{10}$ Leagues. So the Ship running in the firſt Caſe 72 Leagues a Day, will run in the ſecond 28 $\frac{9}{10}$, and in the third 62 $\frac{4}{10}$ Leagues a Day. But beſides the different Courſes, and the different Angles of the Incidence of the Wind upon the Sails, it very often happens that the reſpective Velocities of the Wind againſt the Sails are not equal or the ſame; which muſt cauſe the Ship's Way to increaſe in a certain Ratio that cannot be determined without nearly knowing the Relations of the different Velocities of the Wind. There are ſeveral Ways to find the different relative Velocities of the Wind, laid down by Authors, chiefly by means of a Machine in Form of a Windmill, deſcribed by Mr. *Wolfe*, and improved and brought to Perfection by Mr. *Dons-en-Bray*, of the *Royal Academy*. But for want of theſe Machines, take the following ſimple and eaſy Method of finding the reſpective Velocities of the Wind againſt the Sails.

109. Upon an iron or wooden Rod AB, place a Quadrant CDE, in form of a Weathercock, to whoſe Centre faſten a ſtrong Thread of Silk, ſuſtaining a very round Ball of Wood, or other Matter lighter than Wood. Then it is evident, that the ſtronger the Wind blows, the higher the Ball will riſe, and the Thread CQ will fall upon a greater Number of Degrees of the Quadrant from D towards E.

PROB. IV.

To find the Ratios of the relative Velocities of the Wind.

110. Let us ſuppoſe, when the Wind blows that it raiſes up the Ball F, and the Thread points out the Arch DG; and afterwards, the Wind blowing Fig. 23.

blowing ſtronger, cauſes the Ball to ariſe higher, and the Thread CR points out the Arch DH. Then I ſay, the two Velocities of the Wind will be to one another, as the ſquare Roots of the Tangents of the Arches DG, DH.

DEMONSTRATION.

111. The Ball in the two Poſitions or Situations F and I, is puſhed by the Impreſſion of the Wind, which it receives, in the horizontal Directions FM, IN, at the ſame Time that it deſcends by its own Weight in the vertical ones KF, LI; but it is plain that having taken KF, LI equal, to expreſs the Force whereby the Ball endeavours to deſcend, and having compleated the Parallelograms QM, RN; the horizontal Directions FM, IM will expreſs the Forces of the Impulſe of the Wind upon the Ball. But becauſe of the ſimilar Triangles KFM, CDO, and LIN, CDP, it will be as FM : IN :: DO : DP. Therefore the Forces of the Wind's Impulſe upon the Ball, will be to one another as the Tangent DO of the Arch DG, is to the Tangent DP of the Arch DH. And becauſe the Impulſes are as the Squares of the Velocities, it follows that the Velocities are as the ſquare Roots of the Forces or Impulſes, or as the ſquare Roots of the Tangents DO, DP.

An EXAMPLE.

112. If the Arch DG be 35 Deg. and the Arch DH 45 Deg. their Tangents will be

70020, and 100000; and the ſquare Roots of theſe will be 264, and 316, the neareſt in whole Numbers. Therefore the Velocities of thoſe two Winds will be to one another as 264 to 316, or 66 to 79.

PROB. V.

The Ratio between the Velocities of the Wind being known, to determine thoſe of a Ship upon her different Courſes, and the different Angles of Incidence of the Wind upon the Sails.

113. I ſtill take the three Examples of the firſt Problem, for which having found the Ratios of the Velocities upon the different Courſes, and the Angles of the Incidence of the Wind upon the Sails; if the Velocities of the Wind in all three Caſes, are reſpectively to one another, as the Numbers 66, 79, and 80; the Velocities being by the third Problem 72, $28\frac{9}{10}$, and $62\frac{4}{10}$; we multiply 66 by 72, 79 by $28\frac{2}{10}$, and 80 by $62\frac{1}{10}$, which gives the three Products 4752, $2283\frac{1}{10}$, 4992, expreſſing the Velocities of the Ship; for (by *Art.* 75.) the Velocities of a Ship upon different Courſes under different Angles of Incidence of the Wind againſt the Sails, are in a Ratio compounded of the reciprocal ſubduplicate Ratio of the Diagonals, the ſingle Ratio of the Wind's Incidence upon the Sails, and the ſingle Ratio of the Velocities of the Wind.

114. This is the moſt compound, and at the ſame Time uſeful Problem in the whole Affair of the working of Ships: yet it is really very eaſy and ſimple. For having once found out by an exact Obſervation, how much Way the Ship makes in an Hour, the Angles of the Sails and Keel, and that of the Sails and the Line of the Wind being known, together with the Expreſſion of the Velocity of the Wind; we ſhall have a conſtant Ratio, which will ſerve as the Term of Compariſon in all other Caſes, or Courſes of the ſame Ship, whatever be the Angle of Incidence of the Wind upon the Sails, and its reſpective Velocities. For Example, if we know that a Ship ſails 2 ½ Leagues an Hour, when the Angle of the Sails and Keel is 60 Degrees, and the Angle of Incidence of the Wind upon the Sails 74 Degrees, and the Expreſſion of the Velocity of the Wind 80: and when the Figure of the Ship may be referred to the 15th Table. Then againſt the Angle of the Sails and Keel, 80 Degrees, we take the Expreſſion of the Velocity, which will be 932, and form a Product of theſe three Quantities, *viz.*

Of the Expreſſion of the Velocity———932
Of the Sine of Incidence 74 Degrees, being 96126
And of the Velocity of the Wind——— 80

That Product will be 7167154560, which with the 2 ½ Leagues an Hour, will form a conſtant Term of Compariſon for all the various Courſes of a Ship, under whatſoever Angle of Incidence of the Wind upon the Sails, and with any Velocity of the Wind.

An EXAMPLE.

115. The ſame Ship as above, has ſteered upon a Courſe five Hours, with the Angle of the Sails and Keel 22 Degrees, the Angle of Incidence of the Wind upon the Sails 30 Degrees, and the Expreſſion of the Velocity 90. Now we have theſe three Quantities, *viz.* the Expreſſion 588 of the Velocity found in the Table againſt 22 Degrees, the Sine 50000 of the Angle of Incidence 30 Deg. and the Expreſſion 90 of the Wind's Velocity: the Product of theſe Numbers is 2646000000. And then to obtain the Ship's Way, we ſay, if 7167154560 gives $2\frac{1}{2}$ Leagues an Hour, how many will 2646000000 give? When the Rule is finiſhed, the Ship's Way will be found to be nearly $\frac{92}{100}$ Parts of a League an Hour, and is $4\frac{3}{5}$ Leagues in five Hours.

Another EXAMPLE.

116. The Angle of the Sails and Keel is 51 Degrees, that of the Incidence of the Wind upon the Sails 75 Degrees, and the Expreſſion of the Velocity of the Wind 80. I demand the Way the Ship has run at that Rate in 30 Hours?

The Velocity anſwering to 51 will be 902, the Sine of 75 Degrees 96593, and the Velocity of the Wind 80. The Product of theſe three Numbers is 6970150880, Then we ſay, if 7167154560 gives $2\frac{1}{2}$ Leagues an Hour, how many will 6970150880 give? the Anſwer will be $2\frac{43}{100}$

Leagues an Hour; which makes $72\frac{9}{10}$ Leagues in 30 Hours.

An OBSERVATION.

117. The Problems aforeſaid do ſufficiently ſhew how to compare the Ways of the ſame Ship run upon different Courſes, either to Windward or Large: but though we ſuppoſed a Veſſel to have no other Form than what can be referred to the 6th and 15th Tables, yet it is eaſy to perceive that we muſt work exactly after the ſame Manner for any other Forms of Ships, which may be referred to our ſeveral Tables for the working of Ships: and that in order to compare the Velocities of two or more Ships, we muſt at leaſt know how much better a Sailer the one is than the other.

Amongſt the ſeveral Cauſes that make Veſſels good or bad Sailers, the chief is that which ariſes from their Shape or Mould[*]; this being what we have almoſt entirely remedied in our nine different Tables of Ship-working. It is true, indeed, that ſtrictly ſpeaking, thoſe Tables only ſerve to compare the Velocities of the ſame Ship: for to obtain the exact Ratios of the Velocities of different Ships, Regard ought to be had to their Shape and Magnitude, and even the Proportions of the Magnitudes of their Sails ſhould be taken into Conſideration. I had at firſt a Deſign to do this, and lay down geometrical Rules to determine theſe Proportions, and introduce theſe Conſiderations into our Problems of the working of Ships; but having ſince conſidered, that to obtain this new Degree of Exactneſs, which would ſcarcely be ſenſible in Practice, we ſhouldmake our Problems too complicated, and by that Means have deviated from that Plainneſs which is ſo neceſſary for the Uſe and Practice of Seamen; and beſides, it very rarely happens that the true Shapes of Ships, whoſe Velocities or Ways run one would compare, can be come at.

I ſaid, this new Degree of Exactneſs is very ſeldom ſenſible in Practice, it being eaſy to perceive from our Principles of the working of Ships, that the want of Exactneſs can only ariſe, in part from the Difference between the Ways run by a Ship, reckoned to be a good Sailer, and one that is a heavy Sailer; and, according to Mr. *Guillet*, the beſt Sailer in a Fleet, will out-ſail the worſt only by about three or four Leagues a Day at moſt.

PROB. VI.

The Compariſon of the Velocities of two Ships, when, 1ſt, the Angles of the Sails and Keel are unequal, but the Angles of Incidence of the Wind upon the Sails equal, and the relative Velocities of the Wind equal. 2dly, When the Angles of the Incidence of the Wind upon the Sails are unequal, but the Angles of the Sails and Keel equal, and the Velocities of the Wind equal. 3dly, When the Velocities of the Wind are unequal, but the Angles of the Sails and Keel equal, as alſo

alſo the Angles of Incidence of the Wind upon the Sails equal. 4thly, When the Velocities of the Wind are equal, but the Angles of the Sails and Keel, and the Angles of the Incidence of the Wind upon the Sails unequal. 5thly, When the Angles of the Incidence of the Wind upon the Sails are equal, but the Angles of the Sails and Keel unequal, as alſo the Velocities of the Wind. 6thly and Laſtly, when the Angles of the Sails and Keel are equal, but the Angles of the Incidence of the Wind upon the Sails unequal, and the Velocities of the Wind unequal.

The Reſolution of theſe ſix Caſes do each of them depend upon ſome one Article of the 6th Section, the firſt upon *Art.* 67, the ſecond upon *Art.* 70, the third upon *Art.* 71, the fourth upon *Art.* 72, the fifth upon *Art.* 73, and the ſixth upon *Art.* 74.

An EXAMPLE *of the firſt Caſe.*

119. Since in this Caſe the Angles of the Sails and the Keel are only unequal; the Velocities of two Ships will be to one another (by *Art.* 68.) in the Ratios of the Velocities ſet down in the Tables. So that the Angles of the Sails and the Keel of the firſt being 70 Degrees, and that of the ſecond 65. And alſo, if the Shapes of the Ships be ſuch as the firſt can be referred to the 6th and

15th Tables, and the ſecond to the 9th or 18th, we ſhall find the Number 964 in theſe Tables over-againſt 70 Degrees in the Column of Velocities; which Number will expreſs the Velocity of the firſt Ship; and the Number 946 over-againſt 65 Degrees in the other Table, for the Expreſſion of the Velocity of the ſecond Ship. So that theſe two Velocities are to one another as 964 to 945, or 100 to 98.

Here we may obſerve the ſmall Difference between the two Velocities, although that of the firſt ought to be greater than that of the ſecond, for two Reaſons, *viz.* the Magnitude of the Angles of the Sails and Keel, and the Shape of the Veſſel or Angle of the Prow; ſo that from hence we may obſerve, that a Difference of one or two Degrees in the Poſition of the Sails, as alſo ſome Difference in the Shapes of two Ships, will not produce any ſenſible Error in Practice.

An EXAMPLE *of the ſecond Caſe.*

120. It is manifeſt by the 70th Article, that in this Caſe, if the Shapes of the Ships be ſuch, as they may be referred to the ſame Table, their Velocities will be in the ſingle Ratio of the Sine of Incidence of the Wind upon the Sails: ſo that the Angle of the Incidence of the Wind upon the Sails of the firſt Ship being 60 Deg. and that of the ſecond 70 Deg. the Velocity of the firſt will be to that of the ſecond, as the Sine of 60 Deg. which is 86603 is to the Sine of 70 Deg. which is 93969, or as 100 to 108.

But if the Shapes of the Ships be different, ſo that the firſt being referred to the 6th and 15th Tables, the ſecond may be referred to the 10th and 19th. In this Caſe, in thoſe Tables over-againſt the Angle of the Sails and Keel, being here ſuppoſed the ſame in both Ships, we muſt take the Expreſſions of the Velocities anſwerable to the Angle of the Sails and Keel, in order to make a Ratio compounded of that of the Velocities, and thoſe of the Sines of the Incidence of the Wind upon the Sails. As, the Angle of the Sails and Keel being 44 Deg. for each Veſſel; in the 6th and 15th Tables, over-againſt 44 Deg. we ſhall find the Velocity 874, and in the 10th and 19th Tables, the Velocity 817; now the Angle of Incidence of the Wind upon the Sails of the firſt Ship being 60 Degrees, and that of the ſecond 70; the Velocity of the firſt Ship will be to that of the ſecond, as the Product of 874 by 86603, is to the Product of 817 by 93969, or as 100 is to 101.

An EXAMPE *of the third Caſe.*

121. Since in this Caſe, there are only the relative Velocities of the Wind that are unequal; the Velocities of the Ships will be in the ſame Ratio as that of the relative Velocities of the Wind. If that Ratio be as 100 to 115, the Velocity of the firſt Ship will be to that of the ſecond, as 100 to 115.

But if the Shapes of the Ships be not the ſame, and when the firſt may be referred to the 6th and 15th Tables, the ſecond can to the 10th and

19th: then muſt the Angle of the Sails and Keel be known, (although the ſame in both Ships) that thereby we may obtain the Expreſſions of the Velocities in each Table. If that Angle be 30 Deg. we ſhall find 696 in the 15th Table, under the Column of Velocities, for the firſt Ship, and 682 under that Column in the 19th Table, for the ſecond Ship; and ſo the Velocity of the firſt will be to that of the ſecond, as the Product of 696 by 100, is to the Product of 682 by 115, or nearly as 100 to 113.

An EXAMPLE *of the fourth Caſe.*

122. The Expreſſions of the Velocities of the Wind being equal, if the Angle of the Sails and Keel of the firſt Veſſel be 60 Deg. and that of the ſecond 35 Deg. the Angle of Incidence of the Wind upon the Sails of the firſt Veſſel 77 Deg. and the ſame Angle upon the Sails of the ſecond 70 Deg. If the Shapes of the Veſſels be ſuch that they can be referred, the former to the 6th and 15th Tables, and the latter to the 9th or 18th; then in the Column of Velocities of the 6th or 15 Tables, will be found the Number 932 againſt 60 Degrees, and in the 9th and 18th Tables 740 againſt 35 Degrees. But the Sine of 77 Degrees is 97437, and that of 70 Degrees is 93967. Now (by *Art*. 62.) the Velocity of the firſt Ship will be to that of the ſecond, as the Product of 932 by 97437 is to the Product of 740 by 93969, or nearly as 100 to 76.

An Example *of the fifth Caſe.*

123. The Angles of Incidence of the Wind upon the Sails of two Ships being 70 Degrees, the Angle of the Sails and the Keel of the firſt 81 Deg. and of the ſecond 60 Deg. if the Shapes of the two Ships be ſuch as may be referred to the 6th or 15th Table, the Expreſſion of the Velocities or Diagonals for 81 Degrees, will be found 984, and for 60 Degrees, the Number 932. But if the Velocities of the Wind are to one another as 60 to 70, the Velocity of the firſt Ship will be (by *Art.* 73.) to the Velocity of the ſecond, as the Product of 984 by 60, is to the Product of 932 by 70, or as 59040 is to 65240, or nearly as 100 to 110.

But if the Figures of the Ships be different, ſo that the firſt may be referred to the 6th or 15th Table, and the ſecond to the 10th or 19th, in the 15th Table we ſhall find as above, 984 againſt 81 Degrees, and in the 19th Table, 921 againſt 60 Degrees. If the Velocities of the Wind be the ſame as above, the Velocity of the firſt Ship will be to that of the ſecond, as the Product of 984 by 60, is to the Product of 921 by 70, or as 100 to 109.

An Example *of the ſixth Caſe.*

124. The Angles of the Sails and Keels of two Ships being 83 Degrees, the Angle of Incidence of the Wind upon the Sails of the firſt Ship 87 Degrees, and the Angle of Incidence of the Wind upon the Sails of the ſecond Ship 70 Degrees, the re-

relative Velocity of the Wind upon the firſt Ship 80, and that upon the ſecond 90. It is required to find the Ratio between the Velocities of theſe two Ships. By the 74th Article, the Velocities are in a Ratio compounded of the ſingle Ratio of the Sines of the Incidence of the Wind upon the Sails, and the ſingle Ratio of the Velocities of the Wind; and ſo the Velocity of the firſt Ship will be to that of the ſecond, as the Product of 99863, the Sine of 87 Degrees, by 80, is to the Product of 93969, the Sine of 70 Degrees, by 70. Now theſe Products 7989040, and 8457210, are nearly to one another as 100 is to 106. Conſequently, the Velocity of the firſt Ship will be to that of the ſecond, nearly as 100 is to 106. I ſay nearly, for an entire Exactneſs muſt not be expected. But it is ſtill a great Advantage to come near to an Exactneſs, conſidering the great Number of Irregularities which Seamen meet in the Practice, and of which they are very ſenſible.

In this Example the Shapes of Ships are ſuppoſed to be ſimilar, and ſo may be referred to the ſame Tables. But if they be not, and it is known, for Example, when the firſt may be referred to the 6th or 15th Table, the ſecond may be referred to the 9th or 18th. Then having firſt found the Ratio of the Velocities, as above, in thoſe Tables againſt the Angle of the Sails and Keel, (which we ſtill ſuppoſe to be 83 Degrees) we take the Velocities anſwerable to that Angle; and the Number 988 will be found for the firſt Ship, and 993 for the ſecond; this done, having found for the ſame

Example, that the Velocities are in the Ratio of 100 to 106, we compound thoſe two Ratios, that is, the Velocity of the firſt Ship, will be to that of the ſecond, as the Product of 988 by 100, is to the Product of 993 by 106, or as 98800 is to 105258, or elſe as 100 is to 106 ½.

An OBSERVATION.

125. We have ſaid in the Obſervation aforegoing, that the three firſt Problems and the fifth, do contain Methods to determine, by Means of our Tables, the different Velocities of a Ship, together with the moſt advantageous Diſpoſitions both of the Sails and Keel, when ſhe ſails upon a given Courſe with a given Wind, and in the 6th Problem we have ſhewn how to compare the Velocities or Ways run by two Ships, or one only upon different Courſes, and different Diſpoſitions of the Sails, and that in all Caſes, and without any Regard to the moſt advantageous Diſpoſitions; which probably will not be always choſen by Seamen. However this is, it will always be very eaſy to compare, and even determine, the Ways of Ships, by the Rules laid down in that Problem; yet nevertheleſs, I allow there is no Neceſſity to inſiſt ſo nicely upon putting the Sails into the moſt advantageous Diſpoſitions, for running a given Courſe with a given Wind; becauſe 5 or 6 Deg. more or leſs in the Angle of the Sails and Keel, do not oftentimes diminiſh the Ship's Way $\frac{1}{50}$ Part leſs, than would ariſe if the Sails were preciſely in the moſt advantageous Diſpoſitions, as we are go-

ing to ſhew in the following Problem; and this is a very conſiderable Advantage for the Practice.

PROB. VII.

126. *The Compariſon of the Velocities or Ways of the ſame Ship, when moving with the ſame Wind, with the moſt advantageous Angle of the Sails and Keel, and moving when that ſame Angle is greater or leſs by ſeveral Degrees than the moſt advantageous Angle.*

To perform this Compariſon as exact as may be, we ſhall not omit the Minutes in our Tables. It is evident that the Angle of the Courſe of the Wind and the Keel being given, and the Sails which divide that Angle into two equal Parts, one of which is the Angle of the Sails and Keel, and the other, the Angle of Incidence of the Wind upon the Sails. I ſay, it is evident, if the firſt be aſſumed ſome Number of Degrees at Pleaſure, and it be ſubtracted from the given Angle of the Courſe of the Wind and Keel; we ſhall have the ſecond, or the Angle of Incidence of the Wind upon the Sails, and with theſe two Angles we find the Expreſſion of the Velocity of the Ship, by the 4th Caſe of the 6th Problem. But ſince this Diviſion of the Angle of the Courſe of the Wind and the Keel, is made in our Tables with the greateſt Advantage poſſible; we find, by the ſame Caſe

of

of the 6th Problem, the Expreſſion of the Velocity of the Ship, and compare it with the Expreſſion found by taking the Angles of the Sails and Keel of ſome Magnitude at Pleaſure.

An Example.

Suppoſe the Angle of the Courſe of the Wind and the Keel to be 125 Degrees, and the Shape of the Ship ſuch as to be referred to the 9th and 18th Tables. Then in theſe Tables will be found 54 Deg. 34 Min. for the moſt advantageous Angle of the Sails and Keel, and that of the Sails and the Line of the Wind 70 Degrees 26 Min. which two Angles together make 125 Degrees. If now we aſſume the Angle of the Sails and the Keel greater than the moſt advantageous Angle, as ſuppoſe 65 Deg. 35 Min. the Angle of Incidence of the Wind upon the Sails will be 59 Deg. and 25 Min. it being the Difference between 125 Deg. and 65 Deg. 35 Min. and if that ſame Angle of the Sails and Keel be taken leſs than the moſt advantageous Angle, as ſuppoſe 44 Deg. 10 Min. that of the Incidence of the Wind upon the Sails will be 80 Deg. 50 Min. Now againſt theſe three Angles of the Sails and Keel, in the 18th Table, we take the Expreſſions of the Velocities, *viz.* 888, for the moſt advantageous Angle of 54 Deg. 34 Minutes, 945 for the Angle of 65 Deg. 35 Minutes, and 817 for the Angle of 44 Degrees 10 Minutes. Then we take the Sines of theſe three Angles, *viz.* 94225, the Sine of 70 Deg. 26 Minutes, 86089, the Sine of 59 Deg.

25 Min. and 98723, the Sine of 80 Deg. 50 Minutes.

But the Velocities of the Wind being the ſame, or equal, thoſe of the Ships will be, by the 4th Caſe of the Problem aforegoing, as theſe three Products, 888 by 94225, 945 by 86089, and 817 by 88723. And having multiplied them, we ſhall find that the firſt Product, or the Expreſſion of the Velocity of the Ship, in the Caſe of the moſt advantageous Angle of the Sails and Keel, is greater than either of the other two: but the Difference, as we have already obſerved, is but ſmall, ſince the Velocity of the firſt, or moſt advantageous Caſe, being ſuppoſed 100, that of the ſecond will be 97, and that of the third nearly 96.

Another EXAMPLE.

128. Always taking the ſame Veſſel; if the Angle of the Line of the Wind and the Keel be 64 Degrees, the moſt advantageous Angle of the Sails and Keel will be 23 Degrees 43 Minutes, and conſequently the Angle of Incidence of the Wind upon the Sails 40 Degrees 17 Minutes. Now, if the Angle of the Sails and Keel be ſuppoſed to be made greater, as 31 Degrees 25 Minutes, that of the Wind upon the Sails will be 32 Degrees 35 Minutes. And if the ſaid Angle be made ſmaller, as 13 Degrees 48 Minutes, that of the Wind upon the Sails will be 50 Degrees 12 Minutes. Then in the 18th Table we take the Expreſſions of the Velocities anſwerable to thoſe three Angles of the Sails and Keel, which will be found 606 for the

moſt advantageous Angle 23 Degrees 47 Minutes, 669 for 31 Degrees 25 Minutes, and 373 for 13 Degrees 48 Minutes. Then we take the three Sines of the Angles of Incidence of the Wind upon the Sails, *viz.* 64657 of 40 Degrees 17 Min. 53853 of 32 Degrees 35 Minutes, and 76828 of 50 Degrees 12 Minutes.

Now by the 72d Article, or the 4th Caſe of the Problem aforegoing, the three Velocities of the Ship will be to one another, as the three Products: the firſt being 606 by 64657, the ſecond 699 by 53853, and the third 373 by 76828; when theſe Multiplications be compleated, you will find the firſt Product, or that of the moſt advantageous Situation of the Sails, to be greateſt, and the Velocity of the moſt advantageous Caſe being expreſſed by 100, the ſecond will be 96, and the third 73; upon which I ſubjoin a ſmall Obſervation here, which is, that when the Sails are to be ſet in a Poſition at Pleaſure, it is much better to run the Riſk of making the Angle of the Sails and Keel greater than the moſt advantageous Angle, than leſs, and that for two Reaſons; firſt, when it is made leſs, the Velocity of the Ship is leſſened in too great a Proportion, as appears by the two Examples. And ſecondly, the Lee-Way is augmented, which is a conſiderable Fault in the Art of working a Ship; whereas, if that Angle be made greater, the Lee-Way is leſſened.

PROB.

PROB. VIII.

129. *The Compariſon of the Velocities of two Ships moved with the ſame Wind, when one of them ſails large, and the other cloſe upon the Wind.*

Every body knows that a Wind blowing directly upon the Poop would be beſt, if it could fill all the Sails of a Ship; but ſince in this Caſe the Sails of the hindmoſt Maſts hinder the Wind from falling upon thoſe of the Foremaſts, a Ship will make the greateſt Way when ſhe ſails large, with a good Quarter-Wind. But contrariwiſe, the leaſt when ſhe obtains the moſt Wind, or ſails cloſe upon a Wind. Now to make an exact Compariſon in theſe two Caſes, we firſt ſuppoſe both the Ships to be of the ſame Shape: then ſince they ſail with the ſame Wind, their Velocities are, as in the preceding Problem, in the Ratio compounded of that of the Velocities found in the Tables, anſwerable to the Angles of the Sails and Keel, and the Ratio of the Sines of the Wind's Incidence upon the Sails, ſuppoſing the Shape of the Ships to be ſuch as to be referred to the 18th Table. Moreover, let us ſuppoſe the Courſe of the firſt Ship to be 13 Points from the Wind, (or Line of the Wind) and that of the ſecond 5 Points: then will the firſt Ship ſail large, or with a Quarter-Wind, and the ſecond cloſe upon a Wind. Thus, for Example, the Wind being North-Eaſt, the Courſe ſteer'd by the firſt Ship, will be Weſt by South, and that by the ſecond, North by Weſt, that is, allowing 11 Degrees and 15 Minutes to every Point, the Courſe of the firſt Ship makes an Angle with that of the Wind of 146 Degrees, and that of the ſecond an Angle of 56 Degrees. But in the 18th Table you will find, when the Angle of the Courſe of the Ship and the Wind's is 146 Degrees, the Lee-Way is one Degree. And when that ſame Angle is 26 Degrees, the Lee-Way is 10 Degrees. And ſo that the Lee-Way may make good the Courſe; in the firſt Caſe the Angle of the Courſe of the Wind and Keel muſt be 145 Degrees, and the ſame Angle in the ſecond Caſe muſt be 46 Degrees. Now with the Angle of 145 Degrees, in the firſt Caſe we ſhall find the moſt advantageous Angle of the Sails and Keel to be nearly 67 Degrees; and the Angle of the Incidence of the Wind upon the Sails 78 Degrees; and with an Angle of 46 Degrees, the moſt advantageous one, will be found to be 16 Degrees, and the Angle of Incidence of the Wind upon the Sails 30 Degrees. But the Velocities found in the Table againſt the Angle of the Sails and Keel, is 956 in the firſt Caſe, which (by *Art.* 72.) being multiplied by the Sine of the Incidence 78 Degrees, being 97815, will give a Product that ſhall be the Expreſſion of the Velocity of the firſt Ship, and moreover the Velocity anſwerable to the Angle of the Sails and Keel in the ſecond Caſe is 447, which being multiplied by the Sine of 30 Degrees, or 50000, will give a

Product, being the Expreſſion of the Velocity of the ſecond Ship; but it is eaſy to ſee that theſe Products are to one another, nearly as 100 to 24: conſequently, during the Time the firſt Ship has run 100 Leagues, the ſecond will have run but 24.

But if both the Ships have not the ſame Shape, and when the firſt can be referred to the 14th Table, the ſecond may be referred to the 18th Table. We proceed after the ſame Way as above, with this Difference only, that inſtead of taking the Expreſſion of the Velocity of the firſt Ship, as it ſtands in the Table, we find it by the Problem.

PROB. IX.

130. *To find how much a Ship has fallen to Leeward, after ſhe has run large for ſome Leagues, and to find how much ſhe is got to Windward, after ſhe has ſailed for ſome Leagues near the Wind, or even cloſe hawl'd.*

Although the Line of a Ship's Courſe with the ſame Wind, be not exactly a right Line, but a Curve called the *Loxodromick* Curve, yet as the Diſtances here taken upon that Line are but ſmall; they may be looked upon as right Lines, without any ſenſible Error. This being ſuppoſed, let the Ship H M, drove by the Wind blowing in the Direction V B, ſail upon the Courſe B L: then it is evident that to find how far the Ship is fallen to Leeward Fig. 27.

Leeward after ſhe has run the Diſtance B L, the right Line L R muſt be drawn from the Point L, perpendicular to the Line V B continued; which Perpendicular will terminate the Length B R being the Space required. But if the Courſe of the Wind be u B, and the Ship's Courſe be the Line B l, to find in this Caſe how much the Ship has got to Windward after ſhe has run the Space B l, the Perpendicular L r muſt be let fall upon the Line of the Wind; and B r will be the Length required. And both B R and B r may very eaſily be determined by Trigonometry, for in the right-angled Triangles B R L, B r l, the Angles of the Line of the Wind, and the Ship's Way, being known, as alſo the Lengths run, or the Hypothenuſes B L, B l; the Sides B R, B r wanted, may be found by ſaying, as the Radius is to the Sine Complement of the Angle made by the Line of the Wind and the Ship's Courſe, ſo is the Diſtance ſailed to a fourth Number, expreſſing how much the Ship is fallen to Leeward, or got to Windward.

Note, in the firſt Caſe we take the Angle R B L for the Angle the Line of the Wind makes with the Ship's Courſe.

An EXAMPLE *of the firſt Caſe.*

131. The Angle of the Wind's and Ship's Courſe being 146 Degrees, it is required to find how far ſhe is fell to Leeward, when ſhe has run large 10 Leagues? In this Caſe (according to the Note) we muſt take 34 Degrees, the Comple-

ment of 146 Degrees, and ſay, as the Radius 100000, is to the Sine Complement of 34 Degrees 82904, ſo is 10 Leagues to $8\frac{29}{100}$ Leagues, the Length the Ship has fallen to Leeward when ſailing large.

An EXAMPLE *of the ſecond Caſe.*

132. The Angle of the Wind's and Ship's Courſe being 25 Degrees, I demand how far the Ship is got to Windward, after ſhe has run 10 Leagues upon a Wind?

Say, according to the Rule, as the Radius 100000, is to the Sine Complement of 58 Deg. 52992, ſo is 10 Leagues, to $5\frac{3}{10}$ Leagues, being the Length ſhe is got directly to Windward, after ſhe has run 10 Leagues upon a Wind.

PROB. X.

133. *If there be two Ships plying to Windward, and ſtriving to get the Wind of each other, with this Difference, that the Navigators of one of them are more ſkilful in working her, than thoſe of the ſecond; giving the Keel and Sails the moſt advantageous Diſpoſitions, whilſt thoſe of the other Ship keep her as cloſe as poſſible up to the Wind. To determine how much Advantage the firſt has to get the Wind of the other.*

By the 34th Article, the moſt advantageous Angle the Keel makes with the Line of the Wind, when a Ship, gets moſt to Windward, muſt be about 55 Degrees, and the Angle of Incidence of the Wind upon the Sails 35 Degrees. Theſe, you ſee, are the moſt advantageous Diſpoſitions of the Sails and Keel of the firſt Ship. But the ſecond being willing to keep as cloſe as poſſible upon the Wind, let us ſuppoſe that ſhe carries her Head ſo near to the Wind, as that the Angle made by the Wind's Courſe, and the Keel is 34 Degrees, and the Angle of the Wind upon the Sails 22 Degrees: then by the 72d Article, or the fourth Caſe of the ſixth Problem, the Ratio of the Velocities, or the Ways run by the Ships muſt be found, and afterwards, by the preceding Problem, we may find the Ratio of the Spaces the Ships have got directly to Windward in a given Time.

An EXAMPLE.

When the Shapes of the Ships be ſuch as that they may be referred to the 6th and 15th Tables, over-againſt the Angles of the Wind's Courſe and the Keel, we find the Expreſſions of the Velocities; that of the firſt Ship anſwerable to 55 Deg. is 556, and that of the ſecond againſt 34 Degrees is 394. But the Angles of Incidence of the Wind upon the Sails is 35 Degrees in the firſt Ship, and 22 in the ſecond. This done, we multiply 556 by the Sine of 35 Degrees, which is 57358, and 394 by the Sine of 22 Degrees, which is 37461, and we ſhall have theſe two Products 31891048 and 14759634, for the reſpective Expreſſions of the Velocities or Ways of each Ship, which may be reduced, without any great Error, to the Ratio of 319 to 148. Now to obtain the Ratio of the Ways got to Windward by both the Ships, Regard muſt be had to the Lee-Way, being by our Tables 3 Degrees in the firſt Ship, and ſix Degrees in the ſecond: wherefore the Angle of the Line of the Wind, and the firſt Ship's Courſe, will be 58 Degrees, and the Angle of the Line of the Wind, and the ſecond Ship's Courſe, 40 Degrees. Then we ſay, for the firſt Ship, according to the ſecond Caſe of the preceding Problem, as the Radius 100000, is to the Sine Complement of 40 Degrees 76604; ſo is 148, the Expreſſion of the Way of the ſecond Ship, to 113, the Space ſhe is got to Windward †. Hence it appears, that the Spaces got by the Ships to Windward are, as 169 to 113, or 100 to 67 nearly.

135. By uſing the Logarithms theſe Computations may be much abbreviated; for you need only add together theſe three Logarithms, *viz.* 1. the Logarithm of the Expreſſions of the Velocities found in our Tables. 2dly, the Logarithm of the Sine of the Incidence of the Wind upon the Sails. 3dly, And the Logarithm of the Sine Complement of the Angle of the Line of the Wind, and the Ship's Courſe. Laſtly, having

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† The Author ſeems here to have left out, by Miſtake, the Proportion for finding 169, the Space got directly to Windward by the firſt Ship.

taken away the firſt Figure of the Characteriſtick from that Sum, and what remains will be the Logarithm of the Length the Ship is got to Windward.

The Computation *by the* Logarithms *of the firſt Example, as above.*

1ſt, The Expreſſion of the Velocity of the firſt Ship is 556.

2dly, The Angle of Incidence of the Wind upon the Sails, 35 Degrees.

3dly, The Angle of the Wind and the Ship's Courſe 58 Degrees, and their three Logarithms will be

The firſt, ——————————— 2.74507
The ſecond, ——————————— 9.75859
And the third, ——————————— 9.72421

The Sum of theſe three Logarithms — 22.22787

From whence taking away the firſt Figure 2 of the Characteriſtick 22, and the Remainder 2.22787 is the Logarithm of the Space moved by the firſt Ship, according to the Direction of the Wind, to which correſponds the Number 169, being the ſame as was found by the firſt Method.

Now to get the Length the ſecond Ship is got to Windward, we have firſt the Expreſſion 394 of the Velocity found in the Tables. 2dly, The Angle of Incidence of the Wind upon the Sails 22 Degrees, and the Angle of the Line of the Wind and the Ship's Way 40 Degrees. Their three Logarithms will be,

The firſt, ———————————— 2.59550
The ſecond, ——————————— 9.57357
And the third, —————————— 9.88429

The Sum of theſe three Logarithms — 22.05336

From whence taking away the firſt Figure 2 of the Characteriſtick 22, and the Remainder 2.05336 will be the Logarithm of 113, being the Expreſſion of the Way got by the ſecond Ship to Windward, as found above.

It is ſo eaſy to ſee the Reaſon of this Operation by the Logarithms; that I think there is no need of my ſtaying to explain it.

AN OBSERVATION.

136. The Compariſon made above upon two Ships ſtriving to obtain the Weather Gauge is of itſelf applicable to Veſſels that want to get off from Shore, and out to Sea againſt Wind and Tide, or double a Cape. The ſame Compariſon likewiſe may be made without much Error, upon two hoſtile Fleets, ſtriving to get the Wind of each other.

PROB.

PROB. XI.

137. *To find a Ship's Way when turning her Broad-ſide to Windward.*

In foul Weather, when a Ship cannot hold on a Courſe, or in ſome dangerous Part of the Sea, her Broad-ſide is turned towards the Wind; becauſe then the Reſiſtance ſhe ſuffers from the Water as ſhe paſſes through it, is the greateſt of all, and conſequently her Way retarded the moſt that can be: now if the Ship be ſuppoſed to be puſhed along with the ſame Force of the Wind, whether her Poop be turned, ſo as ſhe ſails before the Wind, or her Broad-ſide faces the Wind, ſo as ſhe makes the moſt Lee-Way: it will be very eaſy to find the Ratio between the Velocities of theſe two Caſes; for (by the 67th *Art.*) we may find the Ratio between the Reſiſtances the Ship finds in cutting the Water with her Side, to that ſhe finds in cutting the Water with her Head; and by the 68th Article, the Velocities are in the reciprocal ſubduplicate Ratio of the Reſiſtances.

An EXAMPLE.

138. If the Form of a Ship can be referred to Segments of 30 Degrees, her Reſiſtance in cutting the Water with her Side, by *Art.* 67. will be to her Reſiſtance in cutting the Water by her Head, as 220 is to 1. And (by *Art.* 68.) we ſay as $\sqrt{220}$ is to $\sqrt{1}$, ſo will the Reſiſtance the Ship

finds in cutting the Water with her Head, be to the Velocity in cutting the Water with her Side.

But in our Tables we ſuppoſe the Velocity of the Ship in cutting the Water with her Head to be 1000, ſo that you ſee in order to obtain the Expreſſion of the lateral Velocity; you need only divide 1000 by the ſquare Root of 220, and we ſhall have nearly 68.

This Expreſſion of the Velocity may be compared with any of thoſe that are in the Table. For if, for Example, one knows in foul Weather, that a Ship ſails 5 Leagues an Hour, when the Angle of the Courſe of the Wind and the Keel is 110 Degrees; ſince the Expreſſion of the Velocity is in this Caſe 881; to find how far the Ship has been drove by the ſame Wind, with her Side towards it, we ſay, if 881 gives 5 Leagues, what will 68 give? The Anſwer is $\frac{340}{881}$ Parts of a League. Therefore a Ship, in the firſt Caſe, running 120 Leagues in 24 Hours; will run only about 9 Leagues in the ſecond: but this Computation is to be looked upon as ſomewhat uncertain: becauſe in foul Weather the Sails are furled and taken in, and the Wind acting very much upon the Body of the Ship, makes it impoſſible to know the Ratio of the Forces that the Ship is drove along with in both thoſe Caſes. Notwithſtanding this, we ought not to neglect the Calculation, becauſe by Means thereof we may obtain Limits which will enlighten the Judgment, and always contribute to our making a more exact Eſtimation than otherwiſe.

PROB. XII.

To determine the different Velocities or Ways of one or more Ships, according to the different Quantities of their Sails.

139. A Ship under Sail is often times obliged to ſlacken or leſſen her Way; either to give ſome other Ship Time to come up with her, or elſe run a leſs Riſk of Diſaſter in ſome dangerous Part of the Sea. And for this Purpoſe the Quantity of Sails are leſſened; and that, by the 76th Article, will leſſen the Velocity of the Ship in the Ratio of the ſquare Roots of the Superficies of the Sails. Let us ſuppoſe, for Example, that whilſt a Ship ſails three Leagues an Hour, her Sails are reefed, ſo as to leſſen their Superficies by one third Part, every thing elſe remaining the ſame; her Velocity in the firſt Caſe will be to that of the ſecond, as the ſquare Root of 3, is to the ſquare Root of 2, or nearly as 5 is to 4: then we ſay, if 5 gives 4, what will 3 Leagues give? The Anſwer will be $2\frac{2}{5}$ Leagues, which will be the Ship's Run, after her Sails are leſſened by $\frac{1}{3}$ Part.

140. But if beſides the Diminution of the Sails, the reſpective Velocity of the Wind upon the Sails be leſſened too; then will the Ship's Ways, by *Art.* 77. be in a Ratio compounded of the ſingle Ratio of the relative Velocities of the Wind, and the ſubduplicate Ratio of the Superficies of the Sails, or as the Products of the reſpective Velocity of the Wind

into the ſquare Roots of the Numbers expreſſing the Superficies of the Sails. For Example, let the relative Velocity of the Wind, in the firſt Caſe be expreſſed by 10, and in the ſecond by 7: the ſquare Root of the Number expreſſing the Superficies of the Sails in the firſt Caſe 5, and that in the ſecond 4; then will the Velocity of the Ship in the firſt Caſe be to her Velocity in the ſecond Caſe, as 10 multiplied by 5, or 50, is to the Product of 7 multiplied by 4, or 28, or as 25 to 14. Therefore if the Ship runs 3 Leagues an Hour in the firſt Caſe, and you want to find how far ſhe will run in the ſecond Caſe, we ſay, if 25 gives 3 Leagues, 14 will give 1 League and $\frac{17}{25}$.

As it is commonly in bad Weather, when the Wind becomes too ſtrong, that the Sails are reefed or furled, and ſo the reſpective Velocity of the Wind is almoſt always greater in the ſecond Caſe; being the Reverſe of the Example we have but now laid down. Let us now ſuppoſe, when the Velocity of the Wind in the firſt Caſe is expreſſed by 3, it is in the ſecond expreſſed by 10, and the ſquare Root of the Number expreſſing the Superficies of the Sails, being in the firſt Caſe denoted by 7, that in the ſecond is 5: then according to theſe Ratios, the Ship's Way in the firſt Caſe is to her Way in the ſecond, as the Product of 3 multiplied by 7, is to the Product of 10 multiplied by 5, or as 21 is to 50. Therefore when a Ship runs 21 Leagues in the firſt Caſe, in ſome certain Time, in the ſecond Caſe ſhe will run 50 Leagues in the ſame Time.

141. It eaſily appears that for the Velocity of the Ship to be the ſame in both Caſes, the ſquare Roots of the Numbers expreſſing the Superficies of the Sails, muſt be reciprocally proportionable to the reſpective Velocities of the Wind. Becauſe then the Products of the ſquare Roots of the Numbers expreſſing the Superficies of the Sails, will be equal in both Caſes. If, for Example, the ſquare Root of the Number expreſſing the Surface of the Sails be 7 in the firſt Caſe, and 5 in the ſecond, and the Expreſſion of the Velocity of the Wind in the firſt Caſe 50, but in the ſecond 70: then will both the Products be 350, and ſo the Velocities of the Ship will be the ſame.

We may likewiſe infer from all this, that when the Surfaces of the Sails are known, as alſo the relative Velocity of the Wind; and the Wind becomes higher, and we really know by the fourth Problem, how much or at what Rate it does ſo, then it will be eaſy to know, how much the Sails muſt be reefed or leſſened, ſo as the Ship may continue on running with the ſame Velocity ſhe had at firſt, or nearly with the ſame Velocity. For the Product of the ſquare Root of the Number expreſſing the Superficies of the Sails, and the Velocity of the Wind in the firſt Caſe being equal to the Product of the Velocity of the Wind into the ſquare Root of the Number expreſſing the Superficies of the Sails in the ſecond Caſe; if the Product in the firſt Caſe be divided by the reſpective Velocity of the Wind in the ſecond, the Quotient will give the ſquare Root of the Number expreſſing the Superficies of the Sails in the ſecond Caſe. As ſuppoſe 8 expreſſes the ſquare Root of the Superficies of the Sails in the firſt Caſe, and 12 be the Velocity of the Wind, and let the Velocity of the Wind in the ſecond Caſe be 16. Then, if 96, the Product of 8 and 12, be divided by 16, the Quotient 6 will expreſs the ſquare Root of the Number expreſſing the Quantity of the Sails abroad in the ſecond Caſe. Therefore the Quantity of the Sails abroad in the firſt Caſe will be to the Quantity abroad in the ſecond, as the Square of 8 *to* the Square of 6, or as 64 to 36, or as 16 is to 9. So that the Sails muſt be contracted the $\frac{9}{16}$ Part of what they were at firſt: or, to make the thing more evident, their Breadth remaining the ſame, and their Depth being 16 Feet, they muſt be drawn up, ſo as to have but 9 Feet in Depth.

142. But moreover, what may be very convenient for Practice, is, that the Superficies of the Sails need not be known, either to determine the Ratio between the Ways of the Ship, or keep her going on at the ſame Rate; for if the Superficies of the Sails in the firſt Caſe be expreſſed by Unity, it is eaſy to perceive by this Method, that the Superficies of the Sails in the ſecond Caſe, will be expreſſed by a Fraction; which will denote the Ratio of the Superficies of the Sails in both Caſes. In the Example above we have 1 for the Surface of the Sails, and 12 for the Velocity of the Wind in the firſt Caſe, and 16 for that in the ſecond: then dividing 12 by 16; and the Fraction $\frac{12}{16}$, or $\frac{3}{4}$ will be the Expreſſion of the ſquare Root of the Sails in the ſecond Caſe; and ſquaring this Fraction

we ſhall have $\frac{9}{66}$, or $\frac{3}{22}$, expreſſing the Superficies of the Sails in the ſecond Caſe, being the ſame as above.

What is here ſaid of one Ship only, is applicable to two or more different Ships, nearly of the ſame Form, carrying more or leſs Sails.

143. If, beſides the Alterations of the Force of the Wind, and the Quantities of the Sails, a Ship does alſo change her Courſe; then Regard muſt be had to the different Direction of the Wind upon the Sails, which increaſes or leſſens the Ship's Way, by the 70th Article, in the Ratio of the Sines of the Wind's Incidence upon the Sails: then to obtain the Ratio of the Velocities of the Ship in both theſe Caſes, we muſt find the compound Ratio of the ſquare Roots of the Numbers expreſſing the Sails, of the relative Velocities of the Wind, and the Sines of Incidence of the Wind upon the Sails. In the firſt Caſe, for Example, let the ſquare Root of the Expreſſion of the Sails be 1, the relative Velocity of the Wind 50, and the Angle of Incidence of the Wind upon the Sails 45 Degrees, its Sine being 70710. And in the ſecond Caſe, the ſquare Root of the Expreſſion of the Sails $\frac{2}{5}$, the relative Velocity of the Wind 80, and the Angle of Incidence of the Wind upon the Sails 30 Degrees, whoſe Sine is 50000: then, by the 79th Article, the Velocities of the Ship will be in a Ratio compounded of theſe three Ratios, or as the Products of theſe three Numbers, *viz.* the Product of 1 multiplied by 50, multiplied by 70710, which is 3535500

to the Product of $\frac{2}{5}$ multiplied by 80, and by 50000, which is 1600000. Therefore the Velocity of the Ship in the firſt Caſe will be to that in the ſecond, as 3535500 is to 1600000, or nearly as 100 to 45.

144. Laſtly, if it happens that the Angle of the Sails and Keel be different in both theſe Caſes; then in the Tables you muſt take the Expreſſions of the Velocities anſwering to the Angles of the Sails and Keel, and make a Ratio compounded of the ſubduplicate Ratio of the Surfaces of the Sails, the ſingle Ratio of the relative Velocities of the Wind upon the Sails, and the Ratio of the Expreſſions of the Velocities found in the Tables: this compound Ratio, by the 79th Article, will be the ſame as that of the Velocities of the Ship. If the Form of the Ship be ſuch as it may be referred to the 6th or 15th Tables, and the Expreſſion of the ſquare Root of the Sails in the firſt Caſe be 1, the relative Velocity of the Wind 50, the Angle of Incidence of the Wind upon the Sails 45 Deg. whoſe Sine is 70710, and the Angle of the Sails and Keel 27 Degrees; againſt which Angle in the 6th and 15th Tables, ſtands 665 for the Velocity: and in the ſecond Caſe, the ſquare Root of the Sails is $\frac{2}{3}$, the relative Velocity of the Wind 80, and the Angle of Incidence of the Wind upon the Sails 39 Degrees, whoſe Sine is 62932, and the Angle of the Sails and Keel 22 Degrees; correſpondent to which we find 588 in the Tables of Velocity. Now it is very eaſy to obſerve, that the Product of the four firſt Numbers, *viz.* 1, 50, 70710, and 665,

is to the Product of the four ſecond, *viz.* $\frac{2}{3}$, 80, 62932, and 588, as 100 to 84. Therefore, if the Ship in the firſt Caſe runs 100 Leagues in ſome given Time, ſhe will run in the ſecond Caſe not more than 84 Leagues in the ſame Time.

It is proper to obſerve, that we muſt not here expect a rigorous Exactneſs; becauſe the Velocity of the Wind being not infinite with regard to that of the Ship, it is certain when her Velocity becomes greater or leſſer, the reſpective Velocity of the Wind upon the Sails does increaſe or decreaſe alſo. But the Quantity thereof will be always very ſmall with regard to the abſolute Velocity of the Wind; and conſequently to the relative Velocity of the Wind upon the Sails, which makes the Error become very ſmall.

PROB. XIII.

To determine the Ways run by a Ship in the three Varieties of ſailing, viz. when ſhe goes before the Wind, goes large, and goes upon the Wind.

145. One of the moſt eſſential things to obtain this Compariſon is the Knowledge of the Quantity of Sails that a Ship bears, in each of the three Caſes of her running. And as that Quantity is almoſt always different, not only in different Ships, but even in the different Ways of ſailing in the ſame Ship; we cannot here make a very exact Compariſon of the Ways run by her: but yet it is

very eaſy for Sea-Officers and Pilots who are aboard a Ship, to know the Quantity of Sails ſhe bears. And to make the Solution of this Problem more evident, let us ſuppoſe a Third Rate Ship of War to go before the Wind.

The Fore-ſail 68 Feet broad and 36 deep, makes 2448 ſquare Feet; the Sprit-ſail 49 Feet broad and 23 deep, makes 1127 ſquare Feet; the main Top-ſail 3455 ſquare Feet; and the Main-Top-gallant-ſail 950 ſquare Feet: the Surfaces of theſe Sails being added together gives 7980 ſquare Feet of Sail, which the Ship carries when ſhe runs before the Wind.

Now here is the Quantity of Sails nearly that the ſame Ship carries, when ſhe runs large, and cloſe upon a Wind.

	ſq. Ft.
The Main-ſail 75 Feet broad, and 42 deep	3150
The Fore-ſail the ſame as above,	2448
The Sprit-ſail the ſame as above,	1127
The Main-top-ſail, and Fore-top-ſail taken together,	5546
The Mizen,	1400
The Mizen-top-ſail	958
The Fore-top-gallant-ſail,	666
The Sum of all theſe Surfaces of Sails, is	16392

We aſſume this Quantity of Sails, as being that which a Ship bears when ſhe goes large, or elſe runs near the Wind.

Now, all things elſe being equal, the Ways run by the Ship, will be to one another, by the preceding Problem, in the ſubduplicate Ratio of the Surfaces of the Sails; ſo that if we extract the ſquare Roots of 7980, and 16392, we ſhall find them to be 89 and 128, nearly. Wherefore, in this Caſe, the Ways run by the Ship will be as 89 to 128.

But when the Angles made by the Sails and Keel is different, we find in our Tables the Expreſſions of the Velocities anſwerable to thoſe ſame Angles. And moreover, ſince the Angles of Incidence of the Wind upon the Sails is different, the Ways run by the Ship will be, by the 72d Article, in the Ratio of the Sines of the ſaid Angles.

Therefore, if we ſuppoſe the relative Velocity of the Wind to be the ſame in all the three Caſes, *viz.* when the Ship ſails before the Wind, runs large, or runs upon the Wind; her Rates of going will, by the 72d Article, be in a Ratio compounded of theſe three Ratios, *viz.* the ſingle Ratio of the ſquare Roots of the Expreſſions of the Sails, the Ratio of the Velocities found in the Tables, and the ſingle Ratio of the Sine of Incidence of the Wind upon the Sails, or as the Products of thoſe three Numbers. An Example will make this more evident.

An EXAMPLE.

146. Becauſe when a Ship goes before the Wind, the Place to which ſhe is to ſail lies directly in the Line or Courſe of the Wind; the Planes or Surfaces

faces of the Sails may be ſet perpendicular to the Line of the Wind, which Surfaces will be perpendicular to the Keel too: conſequently, in this Caſe, the Angle of the Line of the Wind and Keel may be ſaid to be 180 Degrees, and the Angle of the Sails and Keel 90 Degrees, and the Angle of Incidence of the Wind upon the Sails 90 Degrees: therefore when a Ship goes before the Wind, we ſhall have the ſquare Root of the Sails (found above) to be 89.

Now againſt the Angle made by the Sails and Keel, we find the Expreſſion of the Velocity in our Tables to be 1000; and the Sine of the Angle of Incidence of the Wind upon the Sails will be 100000, the Product of theſe three Numbers being 8900000000, will expreſs the Velocity of the Ship when going before the Wind. But here it is proper to obſerve, that the Form of the Ship is ſuch, as that it can be referred to the 15th Table.

Now let the Ship run large, upon a Courſe 13 Points diſtant from that of the Wind; which will make the Angle of the Line of the Wind and Keel, to be 146 Degrees 15 Minutes, or omitting the Minutes, 146 Degrees, ſuppoſing the Lee-Way to be nothing: then in the Table the Angle made by the Sails and Keel will be found about 68 Degrees, and the Angle of Incidence of the Wind upon the Sails 78 Degrees: and ſo in this Caſe the ſquare Root of the Expreſſion of the Sails, found as above, will be 128, and againſt the Angle of the Sails and Keel we ſhall find 956 in the Tables, expreſſing the Velocity, and the Sine of

78 Degrees, the Angle of Incidence of the Wind upon the Sails is 97815, and the Product of theſe three Numbers being 11969425920, will be the Expreſſion of the Velocity of the Ship when ſhe goes large.

Again, let the Ship ſail near the Wind, upon a Courſe diſtant from that of the Wind about 5 Points; that is, let the Angle made by the Courſe of the Ship, and that of the Wind be 58 Degrees; then in the 15th Table we ſhall find, in this Caſe, the Angle of the Sails and Keel to be nearly 20 Degrees, the Angle of Incidence of the Wind upon the Sails, 35 Degrees, and the Expreſſion of the Velocity 556; and ſince in this Caſe, the Lee-Way is found to be 3 Degrees; the Angle made by the Courſe of the Wind and Keel muſt be 55 Degrees. Therefore we have now the three following Numbers, the ſquare Root of the Expreſſion of the Sails, found above to be 128, the Expreſſion of the Velocity 556, and the Sine 57358 of 35 Degrees, being the Angle of the Incidence of the Wind upon the Sails, and their Product 4082054144 will be the Expreſſion of the Velocity of the Ship running to Windward.

But ſince the Expreſſions thus found of the Velocities of the Ship when ſailing before the Wind, large, or upon a Wind, are too great, it will be proper to bring them within a leſs Compaſs. In order to which, let the Velocity when the Ship runs large be expreſſed by 100, then ſhall we have that wherewith ſhe runs before the Wind, by ſaying, if 11969425920 gives 100, what will

890000000 give? the Anſwer will be nearly 74. And to find the Velocity when ſhe runs near the Wind, ſay, if 11969425920 gives 100, what will 4082054144 give? the Anſwer will be nearly 34. So that we have got three other Expreſſions of the Velocities of the Ship, when running before the Wind, large, or upon the Wind, *viz.* 74, 100, and 34, reſpectively. Therefore, if the Ship ſails large 100 Leagues in a given Time, ſhe will run but 74 Leagues before the Wind, and about 34 upon the Wind, in the ſame Time. Which is ſufficiently confirmed by Experience, and evidently proves the Goodneſs of our Rules.

General Rules to find the abſolute Forces of Water and Wind; particularly that of the Water againſt the Prow and Body of a Ship, and of the Wind againſt the Sails.

Since the moſt part of the following Rules may be applied to the working of a Ship, I am of Opinion, that laying them down here will be no Deviation from our Subject: they are beſides very curious and uſeful, and moſt of them even a Continuation of the Uſes of our Tables. But as it is not my Deſign to employ any algebraic Computations in their Uſes; I ſhall content myſelf with giving only the bare Reſults of the algebraic Inveſtigations of the Rules, which are ſo very eaſy that a Perſon of a very ſlender Skill in Algebra, may himſelf alone find them out.

To find the Force of the Water againſt a Surface of a Foot ſquare, when its Velocity is after the Rate of one Foot in a Second.

147. It is a Principle demonſtrated in Hydroſtatics, that the Force of Water moving againſt a plane Superficies, is equal to the Weight of a Solid of Water, whoſe Baſe is the Surface ſtruck againſt, and Heighth that from whence the Water falling down upon it acquires a Velocity equal to that wherewith the Surface was ſtruck. This being granted, to find the Force of the Impulſe of Water, when its Velocity is given; the Height of that Solid muſt be found, whoſe Weight has the Ratio to that of a cubic Foot of Water, which we ſuppoſe to be 70 Pounds, and that Height will expreſs the Force of the Impulſe of the Water. Now ſee how we find the Height of this Solid. It is known by Experience, that a heavy Body, whether ſolid or fluid, as Water, falls perpendicularly down 14 *(Paris)* Feet in one Second; and we know, that if a Body be moved with a uniform Velocity, equal to the Velocity it has acquired at the End of the Fall of 14 *(Paris)* Feet, that uniform Velocity will give 28 Feet in one Second. But it is demonſtrated in moſt Treatiſes of the Motion of Water, that the Velocities of Water falling down, are as the ſquare Roots of the Heights from whence it falls: therefore to find the Height from whence Water muſt fall, ſo as to acquire a Velocity of one Foot in a Second, we ſay, as the Square

of 28, being 784, is to the Square of 1, which is 1; ſo is 14 to the Heighth $\frac{14}{784}$ or $\frac{1}{56}$ ſought. Conſequently the Heighth of the Solid of Water, whoſe Weight gives the Value of the Force of the Impulſe, with its Velocity of one Foot in a Second, will be $\frac{1}{56}$ of a Foot. But the Surface being a Foot ſquare, the Solid will be $\frac{1}{56}$ Part of a cubic Foot of Water, or of 70 Pounds, which gives 1 Pound and $\frac{1}{4}$, or 20 Ounces.

To find the Force of the Wind againſt a Surface of one Foot ſquare, its Velocity being one Foot in a Second.

148. According to the Experiments of Mr. *Marriote*, in order for the Impulſe of the Wind to be equal to that of Water againſt the ſame Surface, or equal Surfaces; the Velocity of the Wind muſt be 24 times greater than that of the Water. But, by the ſecond Article, the Impulſes of Fluids are as the Squares of the Velocities. Therefore the Impulſe of the Wind whoſe Velocity gives one Foot in a Second, is to that of the Wind with a Velocity of 24 Feet in a Second, as the Square of 1, is to the Square of 24, or as 1 is to 576. But ſince the Impulſe of the Water with a Velocity of one Foot in a Second, is equal to that of the Wind of twenty-four Feet in a Second; it is evident, if the Water and Wind had both a Velocity of one Foot in a Second, the Impulſe of the Water would be to that of the Wind againſt the ſame Surface, as 576 is to 1, therefore if the Wind and Water ſtrike

againſt equal Surfaces, with equal Velocities, and under equal Angles of Incidence, the Magnitude of the Stroke of the Water will be 576 times greater than that of the Wind.

The aforeſaid Ratio of 576 to 1, is (as we have obſerved in the *Memoirs of the Academy of* 1729, at page 388.) the ſame as that of one Ounce to one Grain: that is, if the Stroke of Water gives Ounces, that of the Wind will only give Grains, every thing elſe being equal. So the Impulſe of Water upon a Foot ſquare with the Velocity of one Foot in a Second, being 20 Ounces, that of the Wind upon one ſquare Foot, with a Velocity of one Foot in a Second, will be 20 Grains.

149. And thus you ſee the Reaſon of the Conſtruction of my firſt Table of entire Forces, and lateral Forces perpendicular, and parallel to the Direction of the Water, to all the Angles of Incidence, for every 30 Minutes, which is effected by taking the greateſt entire Force 20 Ounces for Water, and 20 Grains for Wind: by which Means this Table equally ſerves for Water or Wind; but to avoid Fractions, inſtead of 20 we have taken 20000; that by ſo doing, the Fractions omitted may become leſs than the 1000th part of an Ounce for Water, and the 1000th part of a Grain for the Wind. By this Table we may find the entire Forces of oblique Strokes, and lateral perpendicular and parallel ones, with extreme Eaſe, as we are going to ſhew in the following Examples.

150. Let a plane Surface of 64 Feet ſquare be ſtruck, under an Angle of Inclination of 40 Degrees,

grees, with a Velocity of 10 Feet in one Second: then to get the whole Force of the Impulſe, we muſt firſt take 8264 over againſt 40 Degrees in the Table, and multiply it by 100, the Square of the Velocity, and we ſhall have 826400; which being multiplied by the Number of the ſquare Feet in the Snrface, *viz.* 64, and the Product 52889600, will be the Value of the Magnitude of the Force, which, by dividing it by 1000, or cutting off the three laſt Figures, may be reduced into Ounces for Water, and into Grains for Wind. We have therefore 52886 Ounces for Water, or 3305 Pounds 9 Ounces and 3/5. But if it be for Wind, we reduce the 52889 Grains into Pounds by the common Method of Reduction; which will be 5 Pounds 11 Ounces 6 Penny-weights, and 42 Grains, expreſſing the Magnitude of the Impulſe of the Wind, upon a Surface of 64 Feet, with a Wind of 10 Feet Velocity in one Second of Time, its Angle of Incidence being 40 Degrees.

151. If inſtead of the whole Force, the lateral Force parallel to the Direction of the Water or Wind be only wanted. Over-againſt 40 Degrees will be found 5312, expreſſing the lateral parallel Force of the Water or Wind upon a Foot ſquare, with a Velocity of one Foot in a Second: but the Velocity being ſuppoſed 10 Feet, and the Magnitudes of the Strokes being as the Squares of the Velocities: it is evident that 5312 muſt be multiplied by 100, the Square of 10, to obtain 531200 for a ſquare Foot; ſo that the Surface being 64 Feet, we muſt multiply 531200 by 64, and the

Product 8396800 being divided by 1000, will give 8396 $\frac{4}{5}$ Ounces for Water, and ſo many Grains for Wind, which reduced into Pounds will be 524 Pounds 12 Ounces $\frac{4}{5}$ for Water, and 14 Ounces 4 Penny-weights and 42 Grains for Wind.

152. The Operation is the ſame for the perpendicular lateral Force; for in the Table over-againſt 40 Degrees, is found 6345, expreſſing the perpendicular lateral Force, which in this Example muſt be multiplied by 100, and the Product by 64, and having cut off the three laſt Figures to the right hand, there will ariſe 40608 Ounces for Water, and the like Number of Grains for Wind.

It is proper to obſerve here, that ſince theſe Computations are founded upon the ſpecific Gravities of Water and Air; thoſe of the Water are much more certain and exact than thoſe of the Wind, becauſe the Weight of Water is much better known than that of the Air: for although we have taken the Ratio of the Weight of Water to that of Air, to be as 576 to 1, according to the Experiments of Mr. *Marriote*, yet the greater Part of Philoſophers make the Air to be much lighter, and that Ratio to be as 8 or 900 to 1, and even ſay it is variable, according as the Air is more or leſs denſe.

I ſhall not here ſtay to explain the ſeveral very uſeful Problems that may be ſolved by our firſt Table, ſuch as computing the Forces of Windmills and Water-mills, *&c.* when the Surfaces of their Sails and Ladles are given.

153. If the Prows of Ships conſiſted of ſeveral plane Surfaces differently inclined, it would be eaſy to know all the Effort of the Reſiſtance of the Water againſt the whole Prow, by taking the lateral Effort parallel to the Direction of the Water or Courſe of the Ship, againſt every ſmall Surface according to its Magnitude, and Inclination, the Velocity of the Ship being known, and then the Sum of all the Efforts, will give the Reſiſtance that the Ship ſuffers in cutting the Water with her Prow, when there is none, or but a very little Lee-Way.

If, on the contrary, the Curvatures of the Prows were regular, and their Surface could be taken for a part of a Conoid, generated by the Revolution of a Curve of a known Nature; then by the Theory of the Impulſes of Fluids againſt curve Surfaces; the whole Force of the Reſiſtance of the Water may be known, as Mr. *Bouguer* has ſhewn how to find, in his Treatiſe of Maſting. This Way is doubtleſs the moſt natural and elegant; but it is liable to two conſiderable Inconveniencies; firſt, the curve Surfaces of the Prows are irregular, and ſecondly, if they were not, we ſhould almoſt always fall upon ſuch Fluxions whoſe Fluents cannot be obtained in finite Terms, and withal would afford but a very confuſed Idea of what is to be found. Therefore the ſhorteſt Way is to ſuppoſe the curve Surfaces of the Prows to be made up of a great Number of little plane Surfaces, either triangular, or of ſome other Figure at Pleaſure; and the Sum of the Efforts againſt all theſe Surfaces, will be nearly equal to the Effort of the Reſiſtance of the Water againſt the whole Prow, or of the Reſiſtance that the Ship finds in cutting the Water.

154. After having found by this Method the Reſiſtance of the Water againſt the whole Prow of the Ship, it will then be very eaſy to find the Magnitude of a plane Surface, againſt which ſuch an Effort ſhall be made as is equal to that made againſt the whole Prow; and then compare that Surface to that of the Plane of the Section of the Fore-part of a Ship cut through perpendicular to the Keel, or ſo much of that Plane as extends from the Keel to the Level of the Water.

We call the plane Surface, againſt which an Effort is made equal to that upon the whole Prow, the *Reduced-Plane*; which it is evident will be ſo much the leſs, with reſpect to the Plane of the Section of the Ship, as the Model or Make of the Prow, approaches the nearer to the curve Surface of the leaſt Reſiſtance in cutting the Water found out by Geometricians, or as the Ship is a prime Sailer.

In common Veſſels the ſaid Reduced-plane is nearly between $\frac{1}{8}$ and $\frac{1}{10}$ of the Section of the Ship, as we hope to make appear better hereafter, but here we ſhall take it to be $\frac{1}{9}$ of the whole Plane. So that a firſt rate Ship of War, being $44\frac{2}{3}$ Feet broad, has the whole Plane of the vertical Section of the Prow perpendicular to the Keel, nearly 695 Feet, the ninth Part whereof, being $76\frac{8}{9}$, or 77 Feet, will expreſs the reduced Plane.

155. The Value of the Reduced-plane being found, it will then be eaſy to know the abſolute Force of the Reſiſtance of the Water againſt the Prow of the Ship, its Velocity being known in the Feet moved in a Second of Time. And the Ship's Velocity in Feet moved in a Second may be found by knowing the Way ſhe makes in an Hour; for if this be a Sea-League, or 2850 *French* Fathoms, the Ship will run 2850 Fathoms, or 17100 Feet, in an Hour, or 3600 Seconds, which will give a Velocity of 4 $\frac{3}{4}$ Feet in a Second. But to ſave the Trouble of Calculation, we have put down the following little Table, wherein you have the Velocities of a Ship in one Second, in Feet and Parts, anſwering to the Velocities in an Hour, for every Quarter of a League.

A Table of the Ways run by a Ship in Feet per Second.

Leagues in one Hour.	Feet in one Second.
0 —— 1/4	1 —— 3/16
0 —— 1/2	2 —— 3/8
0 —— 3/4	3 —— 9/16
1 —— 0	4 —— 3/4
1 —— 1/4	5 —— 15/16
1 —— 1/2	7 —— 1/8
1 —— 3/4	8 —— 5/16
2 —— 0	9 —— 1/2
2 —— 1/4	10 —— 11/16
2 —— 1/2	11 —— 7/8
2 —— 3/4	13 —— 1/16
3 —— 0	14 —— 1/4
3 —— 1/4	15 —— 7/16
3 —— 1/2	16 —— 5/8
3 —— 3/4	17 —— 13/16
4 —— 0	19 —— 0

156.

156. If the Way of a Ship be two Leagues in one Hour, we ſhall find in the Table 9 ½ Feet of Velocity in a Second; but the Impulſe of the Water againſt a Surface of one Foot, with a Velocity of one Foot in a Second, being 20 Ounces, it is evident, that to obtain the whole Force of the Reſiſtance which the Ship ſuffers in cutting the Water with her Prow, we muſt multiply 20 Ounces by the Square of 9 ½, which is $\frac{361}{4}$, and the Product 1805 Ounces, will expreſs the Reſiſtance the Ship ſuffers in cutting the Water, if its Reduced-Plane were one Foot ſquare; but that Plane being 77 Feet, we muſt multiply 1805 by 77, and the Product 138985 Ounces, or 8686 Pounds 9 Ounces, will be the Expreſſion of the whole Force of the Reſiſtance that the Ship finds in cutting the Water. This Force, as we have already obſerved in the 13th Article, is always equal, and directly contrary to that of the Wind againſt the Sails.

157. But the relative Velocity of the Wind ſtriking againſt the Sails being known, together with the Angle of Incidence of the Wind, and the Superficies of the Sails, if you be curious to know nearly the whole Force of the Wind againſt the Sails, without finding the Force of the Water againſt the Prow of the Ship; in the firſt Table you muſt find the whole Force anſwering to the Angle of Incidence of the Wind upon the Sails, and multiply that by the reſpective Velocity of the Wind, and the Product by the Surface of the Sails, and then divide the laſt Product by 1000, and the Quotient will be the whole Force of the

Wind againſt the Sails in Grains, which may be reduced to Pounds.

158. When the Sails of a firſt rate Ship of War are all ſpread; their Surfaces all together are nearly 15480 ſquare Feet: and if the relative Velocity of the Wind againſt the Sails be 30 Feet in a Second, and the Angle of Incidence of the Wind upon the Sails 80 Degrees; we ſhall find 19397 againſt 80 Degrees in the firſt Table, which being multiplied by 900, the Square of 30, and the Product by 15480, the Surfaces of the Sails: and laſtly, having divided this laſt Product by 1000, the Quotient 270432974, will expreſs in Grains the whole Force of the Wind upon the Sails, being 29343 Pounds and 14 Ounces.

Becauſe that Force is much greater than the Reſiſtance of the Water when the Ship runs two Leagues an Hour, it is evident the Velocity of the Ship is greater, every thing elſe remaining the ſame, than we ſuppoſed it to be.

159. Now knowing what Velocity the Ship has, ſo as her Reſiſtance in cutting the Water be 29343 Pounds 14 Ounces, or 469502 Ounces; and ſince we have ſeen above, that the Reſiſtance the Ship finds as ſhe cuts the Water, conſiſts of three Products, *viz.* 20 Ounces, the Reduced-plane 77 Feet, and the Square of the Velocity; it is evident, if the Magnitude of the Reſiſtance the Ship finds as ſhe cuts the Water, which is here taken to be 469502, be divided by 1540, the Product of 77 multiplied by 20; the Quotient $304\frac{61}{70}$, will expreſs the Square of the Velocity of

the Ship in Feet *per* Second. But the ſquare Root of 304 $\frac{61}{70}$, is nearly 17 $\frac{13}{16}$ Feet. Which ſhews from the Table aforeſaid, that the Ship's Way, in this Caſe, is 3 $\frac{1}{2}$ Leagues in one Hour. We doubtleſs find this Velocity to be conſiderable, but it muſt be obſerved, that all the Sails are ſpread, and in this Suppoſition we have taken the Angle of Incidence of the Wind upon the Sails very great.

160. If all this be made more ſimple by omitting the abſolute Force of the Wind upon the Sails, and the Reſiſtance of the Water, it is evident, if the Denſities or ſpecific Gravities of Water and Air were equal, the Product of the Square of the Velocity of the Ship into the Reduced-plane, would be equal to the Product of the Square of the relative Velocity of the Wind, into the Surfaces of the Sails, and that ſuppoſing the relative Velocity of the Wind upon the Sails were alone equal to the Velocity of the Ship; then the Reduced-plane would be equal to the Surfaces of the Sails. But ſince the ſpecific Gravity of the Water is 576 times greater than that of the Air, in order for the Velocity of the Ship to be equal to the reſpective Velocity of the Wind moving againſt the Sails; the Surfaces of the Sails ought to be 576 times greater than that of the Reduced-plane. And in this Caſe, the relative Velocity of the Ship being the ſame as the relative Velocity of the Wind upon the Sails, that Velocity will be equal to one half the abſolute Velocity of the Wind.

161. It is eaſy to conclude from the whole of what has been ſaid, that the Product of the Square of the reſpective Velocity of the Wind, into the Surfaces of the Sails, being divided by 576, will be always equal to the Product of the Ship's Velocity into the Reduced-plane: but ſince the ſquare Roots of equal Numbers are equal, therefore if the Product of the relative Velocity of the Wind into the ſquare Root of the Surfaces of the Sails, be divided by 24; the Quotient will be always equal to the Product of the Velocity of the Ship into the Reduced-plane: conſequently, the ſaid two equal Products may be conſidered as made up of four Quantities, *viz.* the reſpective Velocity of the Wind, the ſquare Root of the Sails divided by 24, or the $\frac{1}{24}$ Part of the Root of the Sails, the Velocity of the Ship, and the ſquare Root of the Reduced-plane. But as theſe four Quantities always make two equal Products, they may be reduced into a Proportion, and three of them being given, the fourth may be found, as you will ſee in the following Rules.

RULES *to find the Velocity of a Ship.*

162. When the Surface of the Reduced-plane, thoſe of the Sails, and the reſpective Velocity of the Wind upon the Sails, are given; the Ship's Velocity may be found by the following Proportion; as the ſquare Root of the Reduced-plane is to the $\frac{1}{24}$th Part of the ſquare Root of the Sails, ſo is the re-

reſpective Velocity of the Wind, to the Velocity of the Ship.

An EXAMPLE.

163. Let the Reduced-plane be 77 Feet, its ſquare Root is nearly 8 $\frac{77}{100}$, or $\frac{877}{100}$, and the Surface of the Sails 15480 Feet; then its ſquare Root 124 $\frac{42}{100}$, or $\frac{12442}{100}$, being divided by 24, gives nearly $\frac{518}{100}$.

If the Velocity of the Wind be 30 Feet in a Second, we ſay, if $\frac{877}{100}$ gives $\frac{518}{100}$, or 877 gives 518, what will 30 give? The Anſwer will be 17 Feet, and about $\frac{3}{4}$; which will be the Velocity of the Ship, and this Velocity in the Table aforeſaid, will be found to give nearly 3 Leagues and $\frac{3}{4}$ an Hour.

RULES *to find the Expreſſion of the Reduced-plane.*

164. When the Velocity of the Ship, the Velocity of the Wind upon the Sails, and the Surface of the Sails are given, the Expreſſion of the Reduced-plane, may be found by this Proportion, being only the Inverſe of that aforegoing, *viz.* as the Velocity of the Ship, is to the reſpective Velocity of the Wind upon the Sails, ſo is the $\frac{1}{24}$ part of the ſquare Root of the Sails, to the ſquare Root of the Reduced-plane.

If the Velocity of the Ship be 3 $\frac{3}{4}$ Leagues an Hour, that Velocity in the Table being 17 Feet $\frac{13}{16}$ in a Second, the relative Velocity of the Wind

30 Feet, and the $\frac{1}{24}$ part of the Root of the Sails $\frac{513}{100}$: we ſay, if 17 $\frac{13}{16}$, or $\frac{283}{16}$, gives 30, what will $\frac{518}{100}$ give? The Anſwer will be nearly $\frac{873}{100}$, being the ſquare Root of the Reduced-plane; the Square of which being between 76 and 77, will be the Value of the Reduced-plane itſelf.

165. When the Surfaces of the Sails, the Reduced-plane, and the Velocity of the Ship are given, it is very eaſy to find the relative Velocity of the Wind upon the Sails: as well as when the relative Velocity of the Wind, that of the Ship, and the Reduced-plane, being given; the Surfaces of the Sails may be found. For in the firſt Caſe we ſay, as the $\frac{1}{24}$ part of the ſquare Root of the Surfaces of the Sails, is to the ſquare Root of the Reduced-plane; ſo is the Velocity of the Ship, to the reſpective Velocity of the Wind; and contrariwiſe, in the ſecond Caſe, we ſay, as the relative Velocity of the Wind, is to that of the Ship, ſo is the ſquare Root of the Reduced-plane to the $\frac{1}{24}$ part of the ſquare Root of the Sails.

166. In order to make the aforeſaid Rules more ſimple, we have ſuppoſed two things, which cannot both happen at the ſame time in the Practice. One of which is, that the Angle of Incidence of the Wind upon the Sails is a right Angle, being the Caſe where the Ship ſails directly before the Wind; and the other, when ſhe carries all her Sails ſpread; but when ſhe runs before the Wind, the Sails of the hindmoſt Maſts, do prevent the Wind from falling upon thoſe of the foremoſt Maſts; and conſequently, when the Angle of In-

cidence of the Wind upon the Sails is a right Angle, the Ship cannot carry all her Sails, but when ſhe does ſo, the Angle of Incidence of the Wind muſt be acute. Let us firſt ſuppoſe the Angle of Incidence of the Wind to be a right Angle, or the Ship to run directly before the Wind: in this Caſe, inſtead of having 15480 Feet of Sails, there are only about 7536 Feet, the ſquare Root of which is nearly $\frac{8681}{100}$; then, as we have before obſerved, that ſquare Root muſt be divided by 24; and the Quotient $\frac{362}{100}$ will be had inſtead of $\frac{518}{100}$: ſo that $\frac{362}{100}$ muſt be put in the preceding Rules inſtead of $\frac{518}{100}$.

If, for Example, we propoſe to find the Velocity of the Ship, when the Surface of the Reduced-plane, thoſe of the Sails, and the reſpective Velocity of the Wind are given: we ſay, as the ſquare Root of the Reduced-plane, ſuppoſed above to be $\frac{877}{100}$, is to the ſquare Root of the Surfaces of the Sails divided by 24, found to be $\frac{362}{100}$, or as 877 is to 362, ſo is the reſpective Velocity of the Wind, which we ſtill make to be 30 Feet in a Second, to the Velocity of the Ship, being about $12\frac{1}{4}$; which Velocity will give in the Table between $2\frac{1}{2}$ and $2\frac{3}{4}$ Leagues in an Hour.

167. Now if the Ship runs with all her Sails abroad, for which Purpoſe the Angle of the Incidence of the Wind upon the Sails is acute; in this Caſe the Action of the Wind upon the Sails will be leſſened in the Ratio of the Square of the Radius to the Square of the Sine of the Angle of the Incidence of the Wind upon the Sails. And

thoſe of a very ſlender Skill in Algebra, will eaſily ſee, that the Product of the Square of the reſpective Velocity of the Wind into the Surfaces of the Sails divided by 576, and by a Fraction whoſe Numerator is the Square of the Sine of Incidence of the Wind upon the Sails, and Denominator the Square of the Radius. I ſay, they will eaſily obſerve, that the Product of thoſe three Quantities, is equal to the Product of the Reduced-plane into the Square of the Velocity of the Ship. If the ſquare Root of each Product be extracted, we ſhall get two new equal Products; the firſt of the reſpective Velocity into the ſquare Root of the Sails divided by 24, and by the Sine of Incidence of the Wind upon the Sails, divided by the Radius; and the ſecond, of the ſquare Root of the Reduced-plane into the Velocity of the Ship. And if theſe two equal Products be brought into an Analogy, one of the Terms of the Proportion being the Product of the $\frac{1}{24}$ part of the ſquare Root of the Sails into the Fraction, whoſe Numerator is the Sine of the Incidence of the Wind upon the Sails, and Denominator the Radius; the other Terms of the Proportion will be all ſingle: for if this here be the firſt, the ſecond Term will be the ſquare Root of the Reduced-plane, the third the Velocity of the Ship, and the fourth the reſpective Velocity of the Wind; three of which being known, the fourth may be found by the Golden Rule.

If, for Example, the Velocity of the Ship be wanted, the ſquare Root of the Reduced-plane being $\frac{877}{100}$, the reſpective Velocity of the Wind upon the Sails 30 Feet, the ſquare Root of the Sails divided by 24, $\frac{518}{100}$, and the Angle of Incidence of the Wind upon the Sails 63 Degrees, whoſe Sine 89100 being divided by the Radius, gives the Fraction $\frac{89100}{100000}$, or $\frac{891}{1000}$: we muſt then multiply $\frac{518}{100}$ by $\frac{891}{1000}$, and the Product is $\frac{461532}{100000}$, and ſay, by the Golden Rule, As the ſquare Root of the reduced Plane $\frac{877}{100}$, is to the Product of the $\frac{1}{24}$ part of the ſquare Root of the Sails, multiplied by the Radius, *viz.* $\frac{461538}{100000}$; ſo is the reſpective Velocity of the Wind, being 30 Feet in one Second, to the Velocity of the Ship, which upon finiſhing the Rule will be found to be about 15 Feet and $\frac{7}{10}$; anſwerable to which in the Table, you will find a little more than 3 Leagues and $\frac{1}{4}$ *per* Hour.

168. When a Ship runs before the Wind, or even a little large, the abſolute Velocity of the Wind is equal to the Sum of the relative Velocities of the Wind upon the Sails, and of the Ship. As in the firſt Example aforegoing, the abſolute Velocity of the Wind, is equal to the relative Velocity of 30 Feet in a Second, plus the Velocity of the Ship 12 Feet $\frac{1}{4}$, or 42 Feet $\frac{1}{4}$. And in the ſecond Example, the abſolute Velocity of the Wind is 30 Feet, plus 15 Feet $\frac{7}{10}$, or 45 Feet $\frac{7}{10}$ in a Second.

But when the reſpective Velocity of the Wind upon the Sails is unknown, when the Ship runs before the Wind; the following Rule will give us the abſolute Velocity of the Wind, when the Velocity of the Ship, the reduced Plane, and the Surfaces of the Sails are given.

As the ſquare Root of the Surfaces of the Sails, is to a Sum made up of 24 times the ſquare Root of the Reduced-plane, plus the ſquare Root of the Sails; ſo is the Velocity of the Ship, to the abſolute Velocity of the Wind.

An Example.

Let the ſquare Root of the Sails be $\frac{8681}{100}$, the ſquare Root of the Reduced-plane $\frac{877}{100}$, and the Velocity of the Ship 12 $\frac{1}{4}$ Feet in a Second.

If the ſquare Root of the Reduced-plane $\frac{877}{100}$ be multiplied by 24; we ſhall have $\frac{21048}{100}$, which being added to the ſquare Root $\frac{8681}{100}$ of the Sails, gives $\frac{29720}{100}$. Then ſay, according to the Rule, As the ſquare Root of the Sails $\frac{8681}{100}$, is to the Sum $\frac{29729}{100}$; ſo is the Velocity of the Ship 12 $\frac{1}{4}$ Feet (in a Second), to the abſolute Velocity of the Wind; which will be found to be about 42 $\frac{1}{4}$ Feet.

If the Ship does not run directly before the Wind, or the Angle of Incidence of the Wind upon the Sails be acute: the abſolute Velocity of the Wind may be found by the following Rule, when the Surfaces of the Sails, that of the Reduced-plane, the Angle of Incidence of the Wind upon the Sails, and the Velocity of the Ship, be all given, *viz.* as the ſquare Root of the Sails, is to the Sum of 24 times the ſquare Root of the Reduced-plane, multiplied by the Radius, and divided by the Sine of the Incidence of the Wind upon the Sails, and the ſquare Root of the Sails; ſo is the Velocity of the Ship, to the abſolute Velocity of the Wind.

An

AN EXAMPLE.

We ſuppoſe in this Example, the Ship to carry much Sail, by reaſon of the Obliquity of the Sails and Wind, and ſo we ſhall here take the ſquare Root of the Sails to be $\frac{12442}{100}$; the ſquare Root of the Reduced-plane $\frac{877}{100}$, the Angle of Incidence of the Wind upon the Sails 63 Degrees; whoſe Sine is 89100, and the Velocity of the Ship 15 $\frac{7}{10}$, or $\frac{157}{10}$: then to get the compound Product, we multiply the ſquare Root of the Reduced-plane $\frac{877}{100}$ by 24, and the Product $\frac{21048}{100}$, by the Radius 100000; and this laſt Product 21048000, being divided by the Sine of Incidence 89100, the Quotient is $\frac{23622}{100}$, to which muſt be added the ſquare Root of the Sails $\frac{12442}{100}$; and the Sum $\frac{36064}{100}$, will be the compound Product. Then ſay, As the ſquare Root of the Sails $\frac{12442}{100}$, is to that compound Product $\frac{36064}{100}$; ſo is the Velocity of the Ship $\frac{157}{100}$, to the abſolute Velocity of the Wind: which will be found to be 45 $\frac{6}{10}$ Feet.

169. If the abſolute Velocity of the Wind be known, and that of the Ship be required; it is evident that the Rules we ought to follow in both Caſes, *viz.* when the Ship runs before the Wind, and when ſhe runs large, are only the Reverſe of thoſe aforegoing. So that to find the Ship's Velocity in the firſt Caſe, we ſay, As the Sum of 24 times the ſquare Root of the Reduced-plane, and the ſquare Root of the Sails, is to the ſquare Root of the Surfaces of the Sails, ſo is the abſolute

lute Velocity of the Wind, to the Velocity of the Ship.

An EXAMPLE.

The compound Sum being found to be $\frac{20729}{100}$, the ſquare Root of the Sails $\frac{8681}{100}$, and the abſolute Velocity of the Wind 42 $\frac{1}{4}$ Feet. We ſay, If $\frac{29729}{100}$ gives $\frac{8631}{100}$, what will 42 $\frac{1}{4}$ give? The Anſwer will be 12 $\frac{1}{4}$ Feet in a Second, being the Velocity of the Ship.

Laſtly, to find the Velocity of the Ship in the ſecond Caſe, we ſay, As the compound Sum above is to the ſquare Root of the Sails, ſo is the abſolute Velocity of the Wind, to the Velocity of the Ship.

An EXAMPLE.

The compound Product found as above is $\frac{36064}{100}$. Then we ſay, As $\frac{36064}{100}$, is to the ſquare Root of the Sails $\frac{12442}{100}$, ſo is the abſolute Velocity of the Wind, ſuppoſed to be 45 $\frac{6}{10}$ Feet, to the Velocity of the Ship, which will be found to be 15 Feet, and about $\frac{7}{10}$.

Theſe laſt Rules may ſerve to find the greateſt Velocities of Ships, which in our Tables we have expreſſed by 1000.

An OBSEERVATION.

We might have here added ſeveral more Rules, but thoſe we have given already, are enough to ſhew the Uſe of our Tables.

I foreſee that Seamen will find it difficult to ſet their Sails according to the Angles denoted in the Tables; but I am perſuaded thoſe who love their Art, will always prefer exact Solutions to a random Practice; and, by ſome Means or other, find out Ways of putting the Yards and Keel of a Ship nearly into ſuch Situations, as may be anſwerable to the moſt advantageous Angles in the Tables for her ſailing upon a given Courſe. I ſay nearly, for it would be much to ſet them always ſo as to come within 4 or 5 Degrees of thoſe Angles. If there ſtill remain other Difficulties, I ſhall in time to come ſhew how to ſurmount them, after having made the Trials upon the Ships for this purpoſe. The thing of the moſt Conſequence is, how to know exactly the Angle made by the Line of the Wind and the Keel of a Ship. Becauſe, as we have ſeen, that Angle ſerves as the Foundation of the Uſe of the Tables, as well as the Reſolution of almoſt all the Queſtions in the Art of the working of Ships. And ſince the Ways uſed by Pilots to find that Angle, are not ſufficiently exact, Mr. *Dons-en-bray* of the Royal Academy always buſied in promoting Arts and Sciences, has contrived a very ſimple Machine for this Purpoſe, whereon are ſhewn that Angle at all times, as well as the Angle of the Meridian of the Compaſs, and the Line of the Wind, and the Angle of the Meridian of the Compaſs and the Keel of the Ship. The Deſcription of which Inſtrument may be ſeen in the Memoirs of the Academy for the Year 1731. This Gentleman, moreover, found out ſeveral Inſtruments, ſhewing how to find at any time the Name and Velocity, or relative Force of the Wind; all different from that mentioned in the 110th Article, which he deſigns to make public.

I ought, moreover, to give notice, that if the Forms of the Ships upon which my Tables are conſtructed, be found to differ too much from the real ones of the ordinary Ships as now built. I ſhall hereafter take the horizontal Sections of the Hulls of Ships themſelves, and upon them conſtruct new Tables, which may be added to theſe; thinking myſelf very happy, if my Endeavours can at all contribute towards bringing the Art of Navigation to Perfection.

THE END.

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FOR THE

WORKING of SHIPS.

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Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.	Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.
0. 30	$0\frac{523}{1000}$	$0\frac{65}{100}$	$1\frac{523}{1000}$	15. 30	1428	381	1376
1	6	$0\frac{4}{5}$	6	16	1520	419	1461
1. 30	14	$0\frac{9}{10}$	14	16. 30	1613	459	1547
2	24	$0\frac{98}{100}$	24	17	1714	500	1635
2. 30	38	$0\frac{1}{20}$	38	17. 30	1809	544	1725
3	55	3	55	18	1910	590	1816
3. 30	75	$4\frac{53}{100}$	74	18. 30	2014	539	1909
4	97	7	97	19	2120	690	2004
4. 30	123	10	123	19. 30	2264	744	2101
5	154	13	152	20	2339	800	2198
5. 30	184	18	183	20. 30	2453	859	2298
6	218	23	217	21	2569	921	2398
6. 30	256	29	255	21. 30	2688	986	2498
7	297	36	295	22	2807	1051	2543
7. 30	341	44	338	22. 30	2929	1121	2706
8	388	54	384	23	3053	1193	2811
8. 30	437	65	432	23. 30	3180	1268	2916
9	489	76	483	24	3309	1345	3023
9. 30	545	90	538	24. 30	3439	1426	3130
10	603	105	594	25	3552	1510	3238
10. 30	664	121	653	25. 30	3707	1595	3345
11	728	139	715	26	3844	1685	3454
11. 30	795	159	779	26. 30	3982	1777	3563
12	864	180	865	27	4122	1871	3673
12. 30	937	203	915	27. 30	4264	1969	3782
13	1012	227	986	28	4408	2069	3892
13. 30	1090	254	1060	28. 30	4554	2173	4002
14	1170	284	1136	29	4701	2279	4111
14. 30	1254	314	1211	29. 30	4850	2388	4221
15	1340	346	1294	30	5000	2500	4330

TABLE I.

A Table of the Oblique Strokes or Impulſes of Water or Wind upon a ſquare Foot of Surface, for all the Angles of Incidence of every 30 Minutes, made with a Velocity of one Foot in a Second, when the greateſt perpendicular Force is repreſented by 20000.

Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.	Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.
30. 30	5152	2615	4439	45. 30	10175	7257	7131
31	5304	2732	4547	46	10349	7444	7189
31. 30	5460	2853	4655	46. 30	10523	7633	7244
32	5616	2976	4763	47	10698	7824	7296
32. 30	5774	3102	4870	47. 30	10871	8015	7345
33	5933	3231	4976	48	11045	8208	7390
33. 30	6093	3363	5081	48. 30	11219	8403	7434
34	6254	3497	5185	49	11392	8590	7478
34. 30	6416	3634	5288	49. 30	11564	8793	7510
35	6580	3774	5387	50	11736	8990	7544
35. 30	6744	3916	5491	50. 30	11099	9189	7575
36	6910	4161	5590	51	12079	9387	7602
36. 30	7077	4209	5689	51. 30	12249	9586	7626
37	7244	4360	5785	52	12419	9786	7646
37. 30	7412	4512	5881	52. 30	22588	9987	7664
38	7581	4667	5974	53	12757	10188	7677
38. 30	7751	4825	6066	53. 30	12924	10389	7687
39	7921	4985	6156	54	13090	10591	7694
39. 30	8092	5147	6244	54. 30	13256	10792	7698
40	8263	5312	6345	55	13420	10993	7697
40. 30	8436	5478	6414	55. 30	13584	11195	7694
41	8698	5648	6496	56	13746	11396	7687
41. 30	8781	5832	6577	56. 30	13908	11597	7676
42	8955	5992	6655	57	14067	11798	7662
42. 30	9128	6167	6730	57. 30	14226	11999	7644
43	9302	6344	6803	58	14384	12198	7622
43. 30	9477	6523	6874	58. 30	14540	12297	7597
44	9651	6705	6942	59	14695	12598	7569
44. 30	9825	6887	7008	59. 30	14848	12794	7536
45	10000	7071	7069	60	15000	12991	7500

TABLE I.

A Table of the oblique Strokes or Impulſes of Water or Wind upon a ſquare Foot of Surface, for all the Angles of Incidence of every 30 Minutes, made with a Velocity of one Foot in a Second, when the greateſt perpendicular Force is repreſented by 20000.

Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.	Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.
60. 30	15151	13187	7461	68	17194	15942	6441
61	15299	13324	7416	68. 30	17314	16109	6345
61. 30	15447	13575	7370	69	17432	16274	6247
62	15592	13767	7320	69. 30	17547	16436	6145
62. 30	15736	13958	7266	70	17660	16595	6040
63	15878	14147	7208	70. 30	17772	16753	5932
63. 30	16018	14335	7147	71	17880	16906	5822
64	16157	14522	7082	71. 30	17987	17057	5707
64. 30	16294	14706	7014	72	18091	17206	5463
65	16428	14889	6943	72. 30	18192	17350	5471
65. 30	16560	15073	6866	73	18291	17492	5337
66	16692	15248	6788	73. 30	18387	17630	5222
66. 30	16820	15426	6692	74	18480	17764	5094
67	16940	15599	6622	74. 30	18572	17896	4963
67. 30	17071	15771	6532	75	18660	18024	4830

TABLE I.

A Table of the oblique Strokes or Impulſes of Water or Wind upon a ſquare Foot of Surface, for all the Angles of Incidence of every 30 *Minutes, made with a Velocity of one Foot in a Second, when the greateſt perpendicular Force is repreſented by* 20000.

Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.	Angles of Inclination.	The whole Force.	Lateral parallel Force.	Lateral perpendicular Force.
75. 30	18446	18149	4694	83	19703	19556	2401
76	18829	18270	4555	83. 30	19744	19617	2235
76. 30	18910	18387	4414	84	19781	19673	2067
77	18988	18501	4271	84. 30	19817	19726	1899
77. 30	19063	18611	4126	85	19848	19772	1730
78	19135	18717	3979	85. 30	19877	19816	1560
78. 30	19205	18821	3829	86	19903	19854	1388
79	19272	18918	3677	86. 30	19925	19894	1216
79. 30	19334	19011	3524	87	19945	19917	1044
80	19397	19102	3368	87. 30	19963	19943	870
80. 30	19455	19188	3211	88	19981	19999	697
81	19511	19270	3052	88. 30	19986	19984	522
81. 30	19563	19348	2892	89	19994	19990	349
82	19612	19421	2730	89. 30	19997	19997	174
82. 30	19659	19491	2566	90	20000	20000	0

TABLE II.

A Table of the Lateral Forces of the Strokes of Fluids both perpendicular and parallel to their Direction upon all the Arches of a Quadrant for every 30 *Minutes, expreſſed in Parts of the Radius, which we ſuppoſe to be* 100000.

The Arch of a Circle.		The lateral parallel Force.	The lateral perpendicular Force.
Deg.	Min.	Integers & Fractions.	Integers & Fractions.
0.	30	$0\frac{3}{20000}$	$0\frac{11}{5000}$
1		$0\frac{1}{40}$	$0\frac{7}{50}$
1.	30	$0\frac{3}{250}$	$0\frac{3}{5}$
2		$0\frac{2}{25}$	$1\frac{2}{5}$
2.	30	$0\frac{4}{25}$	$2\frac{4}{5}$
3		$0\frac{7}{25}$	$4\frac{3}{4}$
3.	30	$0\frac{7}{25}$	$7\frac{1}{2}$
3		$0\frac{3}{5}$	$11\frac{1}{3}$
4.	30	$0\frac{19}{20}$	$16\frac{1}{10}$
4		$1\frac{9}{20}$	$22\frac{7}{100}$
5.	30	$1\frac{3}{5}$	$29\frac{1}{3}$
6		3	$38\frac{7}{100}$
6.	30	$4\frac{1}{10}$	$48\frac{1}{4}$
7		$5\frac{1}{2}$	$60\frac{1}{3}$
7.	30	$7\frac{1}{3}$	$74\frac{6}{50}$
8		$9\frac{4}{10}$	$89\frac{43}{50}$
8.	30	$12\frac{1}{50}$	$107\frac{2}{3}$
9		$15\frac{1}{10}$	$124\frac{3}{10}$
9.	30	$18\frac{2}{3}$	$149\frac{11}{123}$
10		$23\frac{3}{50}$	$174\frac{1}{2}$
10	30	$27\frac{22}{20}$	$201\frac{2}{5}$
11		$33\frac{1}{2}$	$231\frac{1}{2}$
11	30	$40\frac{3}{100}$	$264\frac{1}{10}$
12		$47\frac{1}{3}$	$299\frac{3}{5}$
12	30	$55\frac{2}{3}$	338
13		$65\frac{3}{15}$	$379\frac{1}{3}$
13	30	$75\frac{2}{3}$	424
14		$87\frac{1}{3}$	472
14	30	$99\frac{9}{10}$	$523\frac{1}{3}$
15		$114\frac{2}{3}$	$577\frac{2}{3}$
15	30	$130\frac{1}{3}$	636
16		$148\frac{1}{7}$	$696\frac{2}{3}$
16	30	$167\frac{2}{3}$	$763\frac{2}{3}$
17		$188\frac{1}{5}$	833
17	30	$210\frac{2}{3}$	$906\frac{1}{3}$
18		$235\frac{2}{3}$	$983\frac{2}{3}$
18	30	263	1067
19		$291\frac{2}{5}$	$1150\frac{1}{3}$
19	30	323	1230
20		$356\frac{2}{5}$	$1333\frac{2}{3}$
20	30	$391\frac{1}{3}$	$1431\frac{2}{3}$
21		$431\frac{2}{5}$	1534
21	30	473	1341
22		$517\frac{2}{5}$	1759
22	30	$564\frac{1}{3}$	1868
23		$615\frac{3}{10}$	$1988\frac{1}{3}$
23	30	$669\frac{1}{3}$	$2113\frac{1}{3}$
24		$725\frac{2}{3}$	2243
24	30	$185\frac{1}{3}$	2377
25		$850\frac{1}{3}$	2516
25	30	918	$2659\frac{2}{3}$

TABLE II.

A Table of the Lateral Forces of the Strokes of Fluids both perpendicular and parallel to their Direction upon all the Arches of a Quadrant for every 30 Minutes, expreſſed in Parts of the Radius, which we ſuppoſe to be 100000.

The Arch of a Circle.		The lateral parallel Force.	The lateral perpendicular Force.	The Arch of a Circle.		The lateral parallel Force.	The lateral perpendicular Forces.
Deg.	Min.	Integers & Fractions.	Integers & Fractions.	Deg.	Min.	Integers & Fractions.	Integers & Fractions.
26		989⅓	2808	38	30	4382⅔	8041⅓
26	30	1065⅓	2961	39		5605⅓	8347⅓
27		1145⅔	3119	39	30	4818	8580⅓
27	30	1229	3281⅔	40		5046	8852⅔
28		1316⅓	3449	40	30	5281⅔	9133⅔
28	30	1408⅔	3621⅓	41		5525	9412⅓
29		1506⅓	3789⅓	41	30	5774⅔	9697⅔
29	30	1608⅓	3980	42		6032	9986⅓
30		1715	4168	42	30	6306⅔	10278⅓
30	30	1826⅔	4356	43		6570⅔	10573⅔
31		1943	4554	43	30	6850⅔	10873
31	30	2064⅓	4754⅔	44		7140	11173⅓
32		2192	4962⅓	44	30	7436⅓	11478
32	30	2324	5170⅓	45		7741⅔	11785
33		2429⅔	5385⅕	45	30	8053⅓	12012
33	30	2612⅙	5604⅔	46		8374	12407⅓
34		2756⅓	5828⅔	46	30	8702	12722⅓
34	30	2911⅔	6057	47		9061⅔	13039⅔
35		3073⅔	6290	47	30	9385	13359
35	30	3241	6527⅔	48		9741⅔	13680⅓
36		3415	6738	48	30	10058	14003⅔
36	30	3595⅓	7015⅓	49		10474	14329⅓
37		3785⅔	7265⅔	49	30	10852⅓	14656
37	30	3976	7521⅓	50		11240⅔	14984⅔
38		4176⅓	7778⅔	50	30	11468⅔	15314

TABLE II.

A Table of the Lateral Forces of the Strokes of Fluids both perpendicular and parallel to their Direction upon all the Arches of a Quadrant for every 30 *Minutes, expreſſed in Parts of the Radius, which we ſuppoſe to be* 100000.

The Arch of a Circle.		The lateral parallel Force.	The lateral perpendicular Forces.	The Arch of a Circle.		The lateral parallel Force.	The lateral perpendicular Force.
Deg.	Min.	Integers & Fractions.	Integers & Fractions.	Deg.	Min.	Integers & Fractions.	Integers & Fractions.
51		12042$\frac{1}{3}$	15645$\frac{1}{3}$	63	30	25908	23892
51	30	12455$\frac{1}{3}$	15977$\frac{2}{3}$	64		25637$\frac{2}{3}$	24202$\frac{2}{3}$
52		12879$\frac{2}{3}$	16309$\frac{2}{3}$	64	30	26273$\frac{1}{3}$	24510
52	30	13309	16645	65		26921	24814$\frac{2}{3}$
53		13847$\frac{1}{3}$	16979$\frac{1}{3}$	65	30	27572	25115$\frac{2}{3}$
53	30	14198	17314$\frac{2}{3}$	66		28236	25414
54		14657$\frac{1}{3}$	11650$\frac{1}{3}$	66	30	28902$\frac{2}{3}$	25608$\frac{1}{3}$
54	30	15123	17986	67		29582	25999
55		15599$\frac{1}{3}$	18322	67	30	30263$\frac{1}{3}$	26286
55	30	19078	18658	68		30958$\frac{2}{3}$	26569
56		16596$\frac{2}{3}$	18993$\frac{1}{3}$	68	30	31655$\frac{2}{3}$	26848
56	30	17076	19323	69		32364	27122$\frac{2}{3}$
57		17087$\frac{2}{3}$	19063	69	30	32862	27393
57	30	18105$\frac{1}{3}$	19997	70		33798$\frac{2}{3}$	27659
58		18635$\frac{1}{3}$	20330$\frac{1}{3}$	70	30	34522$\frac{2}{3}$	27920
58	30	19170$\frac{1}{3}$	20662	71		35258$\frac{2}{3}$	28176$\frac{2}{3}$
59		19717	20993	71	30	35998$\frac{1}{3}$	28428
59	30	20252$\frac{2}{3}$	21322$\frac{2}{3}$	72		36749	28674$\frac{2}{3}$
60		20833$\frac{1}{3}$	21650$\frac{2}{3}$	72	30	37817$\frac{2}{3}$	28916
60	30	21402$\frac{2}{3}$	21977	73		38263	29152
61		21984	22301$\frac{2}{3}$	73	30	39025	29380$\frac{2}{3}$
61	30	22619$\frac{1}{3}$	22624$\frac{1}{3}$	74		39809$\frac{2}{3}$	29607$\frac{2}{3}$
62		23399$\frac{2}{3}$	22944$\frac{2}{3}$	74	30	40576	29827
62	30	23772	23263	75		41362$\frac{2}{3}$	30040$\frac{1}{3}$
63		24387	23573$\frac{1}{3}$	75	30	42148$\frac{2}{3}$	30248

TABLE II.

A Table of the Lateral Forces of the Strokes of Fluids both perpendicular and parallel to their Direction upon all the Arches of a Quadrant for every 30 Minutes, expreſſed in Parts of the Radius, which we ſuppoſe to be 100000.

The Arch of a Circle		The lateral parallel Force.	The lateral perpendicular Force.	The Arch of a Circle.		The lateral parallel Force.	The lateral perpendicular Force.
Deg.	Min	Integers & Fractions.	Integers & Fractions.	Deg.	Min.	Integers & Fractions.	Integers & Fractions.
76		42947	$30450\frac{1}{3}$	83	30	55390	$32994\frac{2}{3}$
76	30	43723	$30646\frac{1}{3}$	84		56252	$32778\frac{1}{3}$
77		44561	$30835\frac{1}{3}$	84	30	$57107\frac{1}{3}$	32875
77	30	$45357\frac{1}{3}$	$31918\frac{2}{3}$	85		57972	$32954\frac{1}{3}$
78		$46174\frac{2}{3}$	$31195\frac{2}{3}$	85	30	58532	33026
78	30	$46990\frac{1}{3}$	$31365\frac{2}{3}$	86		$59702\frac{1}{3}$	$33090\frac{1}{3}$
79		$47817\frac{1}{3}$	$31529\frac{1}{3}$	86	0	$60596\frac{1}{3}$	33147
79	30	$48641\frac{1}{2}$	$31686\frac{2}{3}$	87		61439	$33196\frac{1}{3}$
80		$49475\frac{1}{3}$	31837	87	30	62303	33234
80	30	50236	$31980\frac{2}{3}$	88		$63178\frac{1}{3}$	$33272\frac{1}{3}$
81		$51151\frac{1}{3}$	32117	88	30	$64045\frac{7}{3}$	33299
81	30	$51989\frac{1}{3}$	32247	89		$64921\frac{1}{5}$	33318
82		$52839\frac{2}{3}$	$32369\frac{2}{3}$	89	30	$65789\frac{1}{3}$	$33329\frac{2}{3}$
82	30	53685	32485	90		$66666\frac{7}{3}$	$33333\frac{1}{3}$
83		54573	32601				

TABLE III.

A Table of the beſt Situations of the Sails of a Ship, that either runs upon a Wind, or elſe large, when the Angle formed by the Courſe or Line of the Wind and Keel, is given.

The Angles made by the Wind's and Ship's Courſe or the Keel.	The beſt Angles that the Sails make with the Line of the Wind to go near a Wind.		The moſt advantageous Angle of the Sails and Keel or Courſe, to keep the Wind.		The Angles made by the Courſe of the Wind and the Ship's Courſe or Keel.	The beſt Angles made by the Sails and the Line of the Wind for ſailing large.		The beſt Angles form'd by the Sails and Keel, to ſail from the Wind or fall to Leeward.	
Deg.	Deg.	Min.	Deg.	Min.	Deg.	Deg.	Min.	Deg.	Min.
4	2	40	1	20	176	88	45	87	15
7	4	40	2	20	173	87	43	85	17
10	6	40	3	20	170	86	39	83	21
13	8	39	4	21	167	85	40	81	20
16	10	39	5	21	164	84	40	79	20
19	12	38	6	22	161	83	38	77	22
22	14	38	7	22	158	82	35	75	25
25	16	30	8	30	155	81	34	73	26
28	18	31	9	29	152	80	34	71	26
31	20	26	10	34	149	79	28	69	32
34	22	22	11	38	146	78	22	67	38
37	24	18	12	42	143	77	25	65	35
40	26	12	13	48	140	77	11	62	49
43	28	4	14	56	137	75	5	61	55
46	29	55	16	5	134	73	57	60	3
49	31	47	17	13	131	72	49	58	11
52	33	54	18	6	128	71	25	56	35
55	35	25	19	35	125	70	26	54	34
58	37	12	20	48	122	69	13	52	47
61	39		22		119	67	58	51	2
64	40	17	23	43	116	67	20	48	40
67	42	24	24	36	113	66	9	46	51
70	44	10	25	50	110	64	6	45	54
73	45	46	27	14	107	62	50	44	10
76	47	23	28	37	104	61	29	42	31
79	49	27	29	33	101	59	37	41	23
82	50	35	31	25	98	58	43	39	17
85	52	8	32	52	95	57	16	37	44
88	53	27	34	33	92	56	1	35	59
90	54	44	35	16	90	54	44	35	16

TABLE IV.

A Table of the Poſition of the Sails, Lee-way, Courſes and Velocities of Ships, whoſe Prow makes a curve lined Angle of 20 *Degrees.*

The Angles made by the Sails and Keel.		Angles made by the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg	Min.	Deg.	Min.		
90		0		90		1000	0
41	15	0	30	41	45	817	8
23	23	1		24	23	625	17
16	37	1	30	18	7	502	26
13	10	2		15	10	461	35
10	56	2	30	13	26	412	43
9	30	3		12	30	376	52
8	29	3	30	11	59	346	61
7	45	4		11	45	326	69
7	8	4	30	11	38	302	78
6	49	5		11	49	285	87
6	27	5	30	11	57	270	95
6	15	6		12	15	256	105
6	2	6	30	12	32	244	113
5	49	7		12	49	233	122
5	38	7	30	13	8	223	131
5	29	8		13	29	214	139
5	22	8	30	13	52	205	148
5	13	9		14	13	197	157
5	6	9	30	14	36	190	165
4	57	10		14	57	183	174

TABLE V.

A Table of the Poſition of the Sails, Lee-way, Courſes and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 25 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
90		0		90		1000	0
53	55	0	30	54	25	902	8
34		1		35	53	752	17
25	23	1	30	26	53	635	26
19	39	2		21	39	567	35
15	57	2	30	18	27	506	43
14		3		17	3	468	52
12	25	3	30	15	55	433	61
11		4		15	19	405	69
10	23	4	30	14	53	381	78
9	41	5		14	41	360	87
9	8	5	30	14	38	342	95
8	42	6		14	42	325	105
8	20	6	30	14	50	311	113
8	8	7		15	8	299	122
7	46	7	30	15	16	286	131
7	32	8		15	32	275	139
7	20	8	30	15	50	265	148
7	10	9		16	10	256	157
6	56	9	30	16	26	247	165
6	50	10		16	50	239	174
6	42	10	30	17	12	231	183
6	35	11		17	35	224	191
6	26	11	30	17	56	217	200
6	17	12		18	17	211	209
6	10	12	30	18	40	205	217

TABLE VI.

A Table of the Poſition of the Sails, Lee-way, Courſes and Velocities of Ships, whoſe Prow makes a curve lined Angle of 30 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel. and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
90		0		90		1000	0
63	26	0	30	63	56	947	8
40	58	1		41	58	860	17
34	3	1	30	35	33	741	36
27	8	2		29	8	665	25
23	25	2	30	25	55	603	43
19	27	3		22	27	556	52
17	10	3	30	20	40	517	61
14	29	4		18	29	485	69
14	19	4	30	18	49	459	78
13	7	5		18	7	434	87
12	14	5	30	17	44	413	95
11	34	6		17	34	394	105
11	3	6	30	17	33	378	113
10	32	7		17	32	362	122
10	11	7	30	17	41	349	131
9	52	8		17	52	337	139
9	33	8	30	18	3	325	148
9	17	9		18	17	314	157
9	2	9	30	18	32	304	165
8	52	10		18	52	295	174
8	40	10	30	19	10	286	183
8	29	11		19	29	278	191
8	20	11	30	19	50	270	200
8	12	12		20	12	263	209
8	3	12	30	20	33	256	217
7	54	13		20	54	249	226
7	45	13	30	21	15	248	235
7	40	14		21	46	237	243
7	30	14	30	22		231	255
7	22	15		22	22	225	261

TABLE VII.

A Table of the Poſition of the Sails, Lee-way, Courſes and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 35 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
90		0		90		1000	0
69	56	0	30	70	26	968	8
53	51	1		54	15	894	17
42	36	1	30	44	6	817	26
35	21	2		37	21	751	35
29	26	2	30	31	56	688	43
25	28	3		28	28	639	52
22	32	3	30	26	2	598	61
20	15	4		24	15	562	69
18	23	4	30	22	53	530	78
17	7	5		22	7	505	87
15	55	5	30	21	25	482	95
14	59	6		20	59	461	105
14	11	6	30	21	41	443	113
13	30	7		20	30	426	122
12	58	7	30	20	18	411	131
12	8	8		20	8	391	139
11	50	8	30	19	50	377	148
11	41	9		20	41	372	157
11	22	9	30	20	52	360	165
11		10		21	5	350	174
10	49	10	30	21	19	340	183
10	35	11		21	35	332	191
10	22	11	30	21	52	323	200

TABLE VII.

A Table of the Poſition of the Sails, Lee-way, Courſes, and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 35 *Degrees.*

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
10	10	12		22	10	314	209
9	55	12	30	22	25	306	217
9	50	13		22	50	299	226
9	40	13	30	23	10	292	235
9	30	14		23	30	285	243
9	18	14	30	23	48	279	255
9	13	15		24	13	273	261
9	4	15	30	24	37	266	269
8	56	16		24	59	261	278
8	49	16	30	25	19	256	287
8	41	17		25	41	250	295
8	33	17	30	26	3	245	304

TABLE VIII.

A Table of the Poſition of the Sails, Lee-way, Courſes, and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 40 *Degrees.*

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
90		0		90		1000	0
74	16	0	30	74	46	963	8
61	58	1		62	58	937	17
50	33	1	30	52	3	874	26
42	11	2		44	11	812	35
36	22	2	30	38	52	760	43
31	49	3		34	49	612	52
28	18	3	30	31	48	670	61
25	29	4		29	29	633	69
23	14	4	30	27	44	602	78
21	28	5		26	28	574	87
19	58	5	30	25	28	549	95
18	44	6		24	44	527	105
17	43	6	30	24	13	506	113
16	50	7		23	50	488	122
16	4	7	30	23	34	471	131
15	25	8		23	25	456	139
14	50	8	30	23	20	442	148
14	21	9		23	21	428	157
13	55	9	30	23	25	416	165
13	27	10		23	27	404	174
13	11	10	30	23	41	394	183
12	51	11		23	51	384	191
12	35	11	30	24	5	374	200

TABLE VIII.

A Table of the Poſition of the Sails, Lee-way, Courſes, and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 40 *Degrees,*

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
12	18	12		24	18	365	209
12	5	12	30	24	35	356	217
11	56	13		24	56	348	226
11	53	13	30	25	23	341	235
11	29	14		25	29	334	243
11	17	14	30	25	47	326	255
11	9	15		26	9	319	261
10	58	15	30	26	28	313	269
10	55	16		26	55	307	278
10	39	16	30	27	9	301	287
10	31	17		27	31	295	295
10	22	17	30	27	52	289	304
10	15	18		28	15	285	313
10	6	18	30	28	36	279	321
9	53	19		28	53	274	330
9	32	19	30	29	2	267	339
9	41	20		29	41	264	347

TABLE IX.

A Table of the Poſition of the Sails, Lee-way, Courſes, and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 45 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min		
90		0		90		1000	0
78	5	0	30	78	35	988	8
66	33	1		67	33	956	17
57		1	30	58	30	912	26
49	16	2		51	16	864	35
43	6	2	30	45	36	817	43
37	59	3		40	59	772	52
34	11	3	30	37	41	726	61
31	2	4		35	2	699	69
28	23	4	30	32	53	666	78
25	27	5		30	27	630	87
23	46	5	30	29	16	606	95
22	19	6		28	19	583	105
21	7	6	30	27	37	563	113
20	4	7		27	4	544	122
19	4	7	30	26	34	525	131
18	23	8		26	23	510	139
17	41	8	30	26	11	495	148
17	4	9		26	4	481	157
16	30	9	30	26		466	165
15	59	10		25	59	447	174
15	54	10	30	26	24	435	183
15	30	11		26	30	425	191
15	10	11	30	26	40	415	200

TABLE IX.

A Table of the Poſition of the Sails, Lee-way, Courſes, and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 45 *Degrees.*

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
14	45	12		26	45	405	209
14	34	12	30	27	4	397	217
14	25	13		27	25	390	226
14	4	13	30	27	34	381	235
13	51	14		27	51	373	243
13	38	14	30	28	8	365	255
13	27	15		28	27	359	261
13		16		29		345	278
12	45	17		29	45	333	295
12	27	18		30	27	322	313
12	9	19		31	9	311	330
11	52	20		32	52	301	347
11	35	21		32	35	292	364
11	18	22		33	18	183	382
10	50	23		33	50	280	399
10	23	24		34	23	271	416
9	59	25		36	59	260	433

TABLE X.

A Table of the Poſition of the Sails, Lee-way, Courſes, and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 50 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg	Min.	Deg.	Min.	Deg.	Min.		
90		0		90		1000	0
80	4	0	30	80	34	992	8
70	46	1		71	46	970	17
62	26	1	30	63	56	939	26
57	18	2		57	18	901	35
49	15	2	30	51	45	863	43
42	50	3		45	50	816	52
40	4	3	30	43	34	789	61
36	33	4		40	33	756	69
33	38	4	30	38	8	725	78
31	6	5		36	6	694	87
29	1	5	30	34	31	669	95
27	17	6		33	17	646	105
25	46	6	30	32	16	610	113
24	23	7		31	23	604	122
23	14	7	30	30	44	586	131
21	54	8		29	54	569	139
21	22	8	30	29	52	552	148
20	30	9		29	30	538	157
19	47	9	30	29	17	523	165
19	10	10		29	10	510	174
18	36	10	30	29	6	497	183
18	2	11		29	2	487	191
17	37	11	30	29	7	475	200
17	11	12		29	11	464	209
16	48	12	30	29	18	454	217

TABLE X.

A Table of the Poſition of the Sails, Lee-way, Courſes and Velocities of Ships, whoſe Prow makes a curve lined Angle of 50 Degrees.

The Angles made by the Sails and Keel.		Angles made by the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg	Min.	Deg.	Min.		
16	47	13		29	47	447	226
16	8	13	30	29	38	436	235
15	45	14		29	45	426	243
15	32	14	30	30	2	419	255
15	18	15		30	18	411	261
14	49	16		30	49	397	278
14	24	17		31	34	383	295
14	2	18		32	2	370	313
13	41	19		32	41	359	330
13	21	20		33	21	347	347
13	3	21		34	3	337	364
12	49	22		34	49	327	382
12	28	23		35	28	318	399
12	10	24		36	10	310	416
11	52	25		36	52	301	433
11	36	26		37	36	293	450
11	18	27		38	18	286	467
11	12	28		39	12	279	484
10	43	29		39	43	272	501
10	25	30		40	25	265	518

TABLE

TABLE XI.

A Table of the Poſition of the Sails, Lee-way, Courſes and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 55 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min		
90		0		90		1000	0
81	52	0	30	82	22	995	8
74	5	1		75	5	979	17
66	52	1	30	68	22	957	26
60	24	2		62	24	928	35
54	43	2	30	57	13	898	43
49	51	3		52	51	868	52
45	37	3	30	49	7	835	61
41	59	4		45	59	804	69
38	51	4	30	43	21	775	78
36	11	5		41	11	749	87
33	35	5	30	39	5	725	95
31	54	6		37	54	699	105
30	5	6	30	36	35	678	113
28	31	7		35	31	658	122
27	10	7	30	34	40	639	131
25	56	8		33	56	620	139
25	3	8	30	33	33	606	148
23	58	9		32	58	588	157
23	6	9	30	32	36	574	165
22	20	10		32	20	560	174
21	39	10	30	32	9	548	183
21		11		32		535	191
20	21	11	30	31	51	523	200
19	55	12		31	55	512	209
19	27	12	30	31	57	502	217

TABLE XI.

A Table of the Poſition of the Sails, Lee-way, Courſes and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 55 *Degrees.*

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
19		13		32		492	226
18	37	13	30	32	7	483	235
18	15	14		32	15	474	243
17	54	14	30	32	24	465	255
17	37	15		32	37	457	261
17		16		33		441	278
16	30	17		33	30	426	295
16	14	18		34	14	416	313
15	38	19		34	38	401	330
15	16	20		35	16	389	347
14	47	21		35	47	378	364
14	35	22		36	35	368	382
13	51	23		36	51	359	399
13	57	24		37	57	349	416
13	39	25		38	39	340	433
13	21	26		39	21	331	450
13	4	27		40	4	323	467
12	46	28		40	46	316	484
12	28	29		41	28	309	501
13	6	30		43	6	300	518

TABLE XII.

A Table of the Poſition of the Sails, Lee-way, Courſes, and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 60 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel, and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
90		0		90		1000	0
83	15	0	30	83	45	997	8
76	38	1		77	38	986	17
70	26	1	30	71	56	969	26
64	40	2		66	40	948	35
59	31	2	30	62	1	923	43
54	38	3		57	38	900	52
50	47	3	30	54	17	970	61
47	7	4		51	7	844	69
44	4	4	30	48	34	817	78
41	7	5		46	7	794	87
38	38	5	30	44	8	770	95
36	39	6		42	39	749	105
34	31	6	30	41	1	727	113
32	48	7		39	48	707	122
31	16	7	30	38	46	688	131
29	56	8		37	56	670	139
28	41	8	30	37	11	654	148
27	28	9		36	28	636	157
26	35	9	30	36	5	623	165
25	42	10		35	42	609	174
24	51	10	30	35	21	695	183
24	9	11		35	9	583	191
23	28	11	30	34	58	571	200
22	51	12		34	51	559	209
22	19	12	30	34	49	548	217

TABLE XII.

A Table of the Poſition of the Sails, Lee-ways, Courſes and Velocities of Ships, whoſe Prow makes a curve-lined Angle of 60 Degrees.

The Angles made by the Sails and Keel.		The Lee-ways, or Angles of the Keel and Ship's Courſe.		The Angles made by the Sails and the Ship's Courſe.		The Ratio of the Velocities.	The Quantity of the Lee-way.
Deg.	Min.	Deg.	Min.	Deg.	Min.		
21	45	13		34	45	538	226
21	12	13	30	34	42	528	235
20	46	14		34	46	518	243
20	24	14	30	34	54	510	255
20	2	15		35	2	501	261
19	20	16		35	20	485	278
18	47	17		35	47	469	295
18	11	18		36	11	455	313
17	42	19		36	42	442	330
17	15	20		37	15	429	347
16	51	21		37	51	417	364
16	29	22		38	29	407	382
16	19	23		39	19	396	399
15	46	24		39	46	387	416
15	27	25		40	27	378	433
15	8	26		41	8	369	450
14	49	27		41	49	361	467
14	28	28		42	28	353	484
14	13	29		43	13	345	501
13	54	30		43	54	338	518

TABLE XIII.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 20 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	54	44
	176		88	15	88	45	0		176	995	54	44	54	44
	173		85	17	87	43	0		173	987	54	44	54	44
	170		83	21	86	39	0		170	980	54	44	54	44
1—	167		81	20	85	40	0		167	972	54	44	54	44
	164		79	20	84	40	0		164	965	54	44	54	44
	161		77	22	83	38	0		161	958	54	44	54	44
	158		75	25	82	35	0		158	951	54	44	54	44
2—	155		73	26	81	34	0		155	944	54	44	55	44
	152		71	26	80	34	0		152	937	54	44	54	44
	149		69	32	79	28	0		149	930	54	44	54	44
	146		57	38	78	22	0		146	923	54	44	54	44
3—	143		65	35	77	25	0		143	916	54	44	54	44
	140		62	40	77	11	0		140	909	54	44	54	44
	137		61	55	75	5	0		137	902	54	44	54	44
4—	134		60	3	73	57	0		134	895	54	44	54	44
	131		58	11	72	49	0		131	887	54	44	54	44
	128		56	35	71	25	0		128	880	54	44	54	44
	125		54	34	70	26	0		125	873	54	44	54	44
5—	122		52	47	69	13	0		122	866	54	44	54	44
	119		51	2	67	58	0		119	859	54	44	54	44
	116		48	40	67	20	0		116	852	54	44	54	44
6—	113		46	51	66	9	0		113	845	54	44	54	44
	110		45	54	64	6	0		110	838	54	44	54	44
	107		44	10	62	50	0		117	831	54	44	54	44
	104		42	31	61	29	0		114	824	54	44	54	44
7—	101	30	41	23	59	37	0	30	101	817	55	44	54	44
	99		39	17	58	43	1		98	808	55	44	53	44
	96		37	44	57	16	1		95	800	55	44	53	44
	93		35	59	56	1	1		92	790	55	44	53	44
	91		35	16	45	44	1		90	780	55	44	53	44

TABLE XIII.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 20 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The An-gles of the Wind and Keel.	The Ratio of the Velo-cities.	The beſt Angles of the Rud-der to tack a-bout.		The beſt Angles of the Rud-der for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
8—	89		34	33	53	27	1		88	760	55	44	53	44
	86		32	52	52	8	1		85	754	55	44	53	44
	83		31	25	50	35	1		82	740	55	44	53	44
	80		29	33	49	27	1		79	724	55	44	53	44
9—	77		28	37	47	23	1		76	710	55	44	53	44
	74		27	14	45	46	1		73	696	55	44	53	44
	71		25	50	44	10	1		70	682	55	44	53	44
10—	68		24	36	22	24	1		67	668	55	44	53	44
	65		23	43	40	17	1		64	654	55	44	53	44
	62	30	22		39		1	30	61	640	56	14	53	14
	59	30	20	48	37	12	1	30	58	625	56	14	53	14
11—	56	30	19	35	35	25	1	30	55	598	56	14	53	14
	53	30	18	6	33	54	1	30	52	570	56	14	53	14
	50	30	17	13	31	47	1	30	49	542	56	14	53	14
	47	30	16	5	29	55	1	30	46	515	56	14	53	14
12—	44	30	14	56	28	4	1	30	43	488	56	14	53	14
	42		13	48	26	12	2		40	461	56	44	52	44
	39		12	42	24	18	2		37	446	56	44	52	44
	36		11	38	22	22	2		34	428	56	44	52	44
13—	33	30	10	34	20	26	2	30	31	412	57	14	52	14
	31		9	29	18	31	3		28	476	57	44	51	44
	28	30	8	30	16	30	3	30	25	346	58	14	51	14
	26	30	7	22	14	38	4	30	22	302	59	14	50	14
	24	30	6	22	12	38	5	30	19	270	60	14	64	14

TABLE XIV.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 25 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	45	44
	176		87	14	88	45	0		176	995	54	44	45	44
	173		85	17	87	43	0		173	990	54	44	45	44
1—	170		83	21	86	39	0		170	985	54	44	45	44
	167		81	20	85	40	0		167	980	54	44	45	44
	164		79	20	84	40	0		164	975	54	44	54	44
	161		77	22	83	38	0		161	970	54	44	54	44
	158		75	25	82	35	0		158	965	54	44	54	44
2—	155		73	26	81	34	0		151	959	54	44	54	44
	152		71	26	80	34	0		152	954	54	44	54	44
	149		69	32	79	28	0		149	949	54	44	54	44
3—	146		67	38	78	22	0		146	944	54	44	54	44
	143		65	35	77	25	0		143	938	54	44	54	44
	140		62	40	77	11	0		140	932	54	44	54	44
	137		61	55	75	5	0		137	926	54	44	54	44
4—	134		60	3	73	57	0		134	920	54	44	54	44
	131		58	11	72	49	0		131	914	54	44	54	44
	128		56	35	71	25	0		128	908	54	44	54	44
	125	30	54	34	70	26	0	30	125	902	55	14	54	14
5—	122	30	52	47	69	13	0	30	122	898	55	14	54	14
	119	30	51	2	67	58	0	30	119	888	55	14	54	14
	116	30	48	40	67	20	0	30	116	876	55	14	54	14
6—	113	30	46	51	66	9	0	30	113	855	55	14	54	14
	110	30	45	54	64	6	0	30	110	843	55	14	54	14
	107	30	44	10	62	50	0	30	107	832	55	14	54	14
	104	30	42	31	61	29	0	30	104	820	55	14	54	14
7—	101	30	41	23	59	37	0	30	101	809	55	14	54	14
	98	30	39	17	58	43	0	30	98	797	55	14	54	14
	95	30	37	44	57	16	0	30	95	786	55	14	54	14
	92	30	35	59	56	1	0	30	92	774	55	14	54	14

TABLE XIV.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 25 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
8—	90	30	35	16	54	44	0	30	90	763	55	14	54	14
	88	30	34	33	53	27	0	30	88	752	55	14	54	14
	86		32	52	52	8	1		85	721	55	44	53	44
	83		31	25	50	35	1		82	692	55	44	53	44
	80		29	33	49	27	1		79	673	55	44	53	44
9—	77		28	37	47	23	1		76	654	55	44	53	44
	74		27	14	45	46	1		73	635	55	44	53	44
	71	30	25	50	44	10	1	30	70	618	56	14	53	14
	68	30	24	36	42	24	1	30	67	601	56	14	53	14
10—	65	30	23	43	40	17	1	30	64	584	56	14	53	14
	62	30	22		39		1	30	61	567	56	14	53	14
	59	30	20	44	37	12	2		58	546	56	44	52	44
11—	57		19	35	35	25	2		55	526	56	44	52	44
	54	30	18	6	33	54	2		52	506	56	44	52	44
	51	30	17	13	31	47	2	30	49	494	57	14	52	14
	48	30	16	5	29	55	2	30	46	481	57	14	52	14
12—	45	30	14	56	28	4	2	30	43	468	57	14	52	14
	43		13	48	26	12	3		40	433	57	44	51	44
	40	30	12	42	24	18	3	30	37	405	58	14	51	14
	38		11	38	22	22	4		34	381	58	44	50	44
	35	30	10	34	20	26	4	30	31	360	59	14	50	14
13—	33		9	29	18	31	5		28	311	59	44	49	44
	31	30	8	30	16	30	6	30	25	256	60	14	48	14

TABLE

TABLE XV.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 30 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	54	44
	176		87	15	88	45	0		176	996	54	44	54	44
	173		85	17	87	43	0		173	992	54	44	54	44
	170		83	21	86	39	0		170	988	54	44	54	44
1—	167		81	20	85	40	0		167	984	54	44	54	44
	164		79	20	84	40	0		164	980	54	44	54	44
	161		77	22	83	38	0		161	976	54	44	54	44
	158		75	25	82	35	0		158	972	54	44	54	44
2—	155		73	26	81	34	0		155	968	54	44	54	44
	152		71	26	80	34	0		150	964	54	44	54	44
	149		69	32	79	28	0		149	960	44	44	54	44
3—	146		67	38	78	22	0		146	956	44	44	54	44
	143		65	35	77	25	0		143	952	44	44	54	44
	140	30	62	40	77	11	0	30	140	947	55	14	54	14
	137	30	61	55	75	5	0	30	137	938	55	14	54	14
4—	134	30	60	3	73	57	0	30	134	932	55	14	54	14
	131	30	58	11	72	49	0	30	131	926	55	14	54	14
	128	30	56	35	71	25	0	30	128	920	55	14	54	14
	125	30	54	34	70	26	0	30	125	914	55	14	54	14
5—	122	30	52	47	69	13	0	30	122	908	55	14	54	14
	120		51	2	67	58	1		119	902	55	44	53	44
	117		48	40	67	20	1		116	895	55	44	53	44
	114		46	51	66	9	1		113	888	55	44	53	44
6—	111		45	54	64	6	1		110	881	55	44	53	44
	108		44	10	62	50	1		107	874	55	44	53	44
	105		42	31	61	29	1		104	867	55	44	53	44
	102		41	23	59	27	1		101	860	55	44	53	44
7—	99	30	39	17	58	43	1	30	98	837	56	14	53	14
	96	30	37	44	57	16	1	30	95	810	56	14	53	14
	93	30	35	59	56	1	1	30	92	785	56	14	53	14

TABLE XV.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 30 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The An-gles of the Wind and Keel.	The Ratio of the Velo-cities.	The beſt Angles of the Rud-der to tack a-bout.		The beſt Angles of the Rud-der for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
8—	91	30	35	16	54	44	1	30	90	766	56	14	53	14
	89	30	34	33	53	27	1	30	88	741	56	14	53	14
	87		32	52	52	8	2		85	726	56	44	52	44
	84		31	25	50	35	2		82	711	56	44	52	44
	81		29	33	49	27	2		79	696	56	44	52	44
9—	78		28	37	47	23	2		76	680	56	44	52	44
	75		27	14	45	46	2		73	665	56	44	52	44
	72	30	25	50	44	10	2	30	70	646	57	14	52	14
	69	30	24	36	42	24	2	30	67	625	57	14	52	14
10—	66	30	23	43	40	17	2	30	64	603	57	14	52	14
	64		22		39		3		61	588	57	44	51	44
	61		20	48	37	12	3		58	572	57	44	51	14
	58		19	35	35	25	3		55	556	57	44	51	44
11—	55	30	18	6	33	54	3	30	52	535	58	14	51	44
	52	30	17	13	31	47	3	30	49	517	58	14	51	44
	50		16	5	29	55	4		46	501	58	44	50	14
12—	47		14	56	28	4	4		43	485	58	44	50	44
	44	30	13	48	26	12	4	30	40	459	59	14	50	14
	42		12	42	24	18	5		37	434	59	44	49	44
	40		11	38	22	22	6		34	394	60	44	48	44
	38		10	34	20	26	7		31	362	61	44	47	44
	36	30	9	29	18	31	8	30	28	325	63	14	46	14

TABLE XVI.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 35 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe. and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M	D.	M.	D.		D.	M.	D.	M.
	280		90		90		0		180	1000	54	44	54	44
	176		87	15	88	45	0		176	998	54	44	54	44
	173		85	17	87	43	0		173	996	54	44	54	44
1—	170		83	21	86	39	0		170	993	54	44	54	44
	167		81	20	85	40	0		167	990	54	44	54	44
	164		79	20	84	40	0		164	986	54	44	54	44
	161		77	22	83	38	0		161	983	54	44	54	44
2—	158		75	25	82	35	0		158	980	54	44	54	44
	155		73	26	81	34	0		155	976	54	44	54	44
	152		71	26	80	34	0		152	972	54	44	54	44
	149	30	69	32	79	28	0	30	149	968	55	14	54	14
3—	146	30	67	38	78	22	0	30	146	960	55	14	54	14
	143	30	65	35	77	25	0	30	143	952	55	14	54	14
	141		62	40	77	11	1		140	944	55	44	53	44
	138		61	55	75	5	1		137	936	55	44	53	44
4—	135		60	3	73	57	1		134	928	55	44	53	44
	132		58	11	72	49	1		131	920	55	44	53	44
	129		56	35	71	25	1		128	911	55	44	53	44
	126		54	34	70	26	1		125	902	55	44	53	44
5—	123		52	47	69	13	1		122	894	55	44	53	44
	120		51	2	67	58	1		119	883	55	44	53	44
	117		48	40	67	20	1		116	870	55	44	53	44
	114	30	46	51	66	9	1	30	113	856	56	14	53	14
6—	111	30	45	54	64	6	1	30	110	843	56	14	53	14
	108	30	44	10	62	50	1	30	107	830	56	14	53	14
	105	30	42	31	61	29	1	30	104	817	56	14	53	14
	102	30	41	23	59	37	1	30	101	804	56	14	53	14
7—	100		39	17	58	43	2		98	791	56	44	52	44
	97		37	44	57	16	2		95	778	56	44	52	44
	94		35	59	56	1	2		92	764	56	44	52	44

TABLE XVI.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 35 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe. and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M	D.		D.	M.	D.	M.
	92		35	16	54	44	2		90	751	56	44	52	44
8—	90		34	33	53	27	2		88	736	56	44	52	44
	87	30	32	52	52	8	2	30	85	720	57	14	52	14
	84	30	31	25	50	35	2	30	82	704	57	14	52	14
	81	30	29	33	49	27	2	30	79	688	57	14	52	14
9—	78	30	28	37	47	23	2	30	76	677	57	14	52	14
	76		27	14	45	46	3		73	665	57	44	51	44
	73		25	50	44	10	3		70	652	57	44	51	44
	70		24	36	42	24	3		67	639	57	44	51	44
10—	67	30	23	43	40	17	3	30	64	618	58	14	51	14
	64	30	22		39		3	30	61	598	58	14	51	14
	62		20	48	37	12	4		58	562	58	44	50	44
	59	30	19	35	35	25	4	30	55	548	59	14	50	14
11—	56	30	18	6	33	54	4	30	52	530	59	14	50	14
	54		17	13	31	47	5		49	505	59	44	49	44
	51	30	29	55	16	5	5	30	46	482	60	14	49	14
	49		28	4	14	56	6		43	461	60	44	48	44
12—	47		26	12	13	48	7		40	426	61	44	47	44
	44		24	18	12	42	7	30	37	411	62	14	47	14
	43		22	22	11	38	9		34	372	63	44	45	44
	42	30	20	26	10	34	11		31	332	65	44	43	44

TABLE XVII.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 40 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	54	44
	176		88	15	88	45	0		176	996	54	44	54	44
	173		85	17	87	43	0		173	990	54	44	54	44
	170		83	21	86	39	0		170	985	54	44	54	44
1—	167		81	20	85	40	0		167	980	54	44	54	44
	164		79	20	84	40	0		164	974	54	44	54	44
	161		77	22	83	38	0		161	968	54	44	54	44
	158	30	75	25	82	35	0	30	158	963	55	14	54	14
2—	155	30	73	26	81	34	0	30	155	961	55	14	55	14
	152	30	71	26	80	34	0	30	152	958	55	14	54	14
	149	30	69	32	79	28	0	30	149	954	55	14	54	14
	147		67	38	78	22	1		146	950	55	44	53	44
3—	144		65	35	77	25	1		143	945	55	44	53	44
	141		62	40	77	11	1		140	941	55	44	53	44
	138		61	55	75	5	1		137	937	55	44	53	44
4—	135		60	3	73	57	1		134	926	55	44	53	44
	132		58	11	72	49	1		131	916	55	44	53	44
	129	30	56	35	71	25	1	30	128	906	56	14	53	14
	126	30	54	34	70	26	1	30	125	896	56	14	53	14
5—	123	30	52	47	69	13	1	30	122	885	56	14	53	14
	120	30	51	2	67	58	1	30	119	874	56	14	53	14
	117	30	48	40	67	20	1	30	116	862	56	14	53	14
6—	115		46	51	66	9	2		113	850	56	44	52	44
	112		45	54	64	6	2		110	837	56	44	52	44
	109		44	10	62	50	2		107	825	56	44	52	44
	106		42	31	61	29	2		104	812	56	44	52	44
7—	103		41	23	59	37	2		101	790	57	44	52	44
	100	30	39	17	58	43	2	30	98	784	57	14	52	14
	97	30	37	44	57	16	2	30	95	772	57	14	52	14
	94	30	35	59	56	1	2	30	92	760	57	14	52	14

TABLE XVII.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 40 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The An-gles of the Wind and Keel.	The Ratio of the Velo-cities.	The beſt Angles of the Rud-der to tack a-bout.		The beſt Angles of the Rud-der for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	92	30	35	16	54	44	2	30	90	754	57	44	52	14
8—	91		34	33	53	27	3		88	738	57	44	51	44
	88		32	52	52	8	3		85	712	57	44	51	44
	85		31	25	50	35	3		82	697	57	44	51	44
	82		29	33	49	27	3		79	683	57	24	51	44
9—	79	30	28	37	47	23	3	30	76	670	58	14	51	14
	76		27	14	45	16	3	30	73	657	58	14	51	14
	74		25	50	44	10	4		70	633	58	44	50	44
	71	30	24	36	42	24	4		67	617	58	44	50	44
	68	30	23	43	40	17	4	30	64	602	59	14	50	14
10—	65		22		39		5		61	588	59	44	49	44
	62	30	20	48	37	12	5	30	58	574	60	14	49	14
	60	30	19	35	35	25	5	30	55	549	60	14	49	14
	58	30	18	6	33	54	6	30	52	506	61	14	48	14
11—	56		17	13	31	47	7		49	488	61	44	47	44
	53	30	16	5	29	55	7	30	46	471	62	14	47	44
	51	30	14	56	28	4	8	30	43	442	63	14	46	14
	49	30	13	48	26	12	9	30	40	416	64	14	45	14
	48	30	12	42	24	18	11	30	37	374	66	14	43	14

TABLE XVIII.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 45 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	45	44
	176		87	14	88	45	0		176	999	54	44	45	44
	173		85	17	87	43	0		173	996	54	44	45	44
1—	170		83	21	86	39	0		170	993	54	44	45	44
	167		81	20	85	40	0		167	990	54	44	45	44
	164	30	79	20	84	40	0	30	164	988	55	14	54	14
	161	30	77	22	83	38	0	30	161	983	55	14	54	14
	158	30	75	25	82	35	0	30	158	978	55	14	54	14
2—	156		73	26	81	34	1		155	973	55	44	53	44
	153		71	26	80	34	1		152	968	55	44	53	44
	150		69	32	79	28	1		149	962	55	44	53	44
3—	147		67	38	78	22	1		146	956	55	44	53	44
	144		65	35	77	25	1		143	954	55	44	53	44
	141	30	62	40	77	11	1	30	140	937	56	14	53	14
	138	30	61	55	75	5	1	30	137	929	56	14	53	14
4—	135	30	60	3	73	57	1	30	134	921	56	14	53	44
	132	30	58	11	72	49	1	30	131	912	56	14	53	44
	130		56	35	71	25	2		128	900	56	44	52	14
	127		54	34	70	26	2		125	888	56	44	52	14
5—	124		52	47	69	13	2		122	876	56	44	52	14
	121		51	2	67	58	2		119	864	56	44	52	44
	118		48	40	67	20	2		116	852	56	44	52	44
	115	30	46	51	66	9	2	30	113	842	57	14	52	14
	112	30	45	54	64	6	2	30	110	829	57	14	52	14
6—	109	30	44	10	62	50	2	30	117	817	57	14	52	14
	106	30	42	31	61	29	2	30	114	806	57	14	52	14
	104		41	23	59	37	3		101	794	57	44	51	44
	101		39	17	58	43	3		98	783	57	44	51	44
7—	98		37	44	57	16	3		95	772	57	44	51	44
	95		35	59	56	1	3		92	756	57	44	51	44

TABLE XVIII.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 45 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	93	30	34	16	54	44	3	30	90	740	58	14	51	14
8—	91	30	34	33	53	27	3	30	88	726	58	14	51	14
	89		32	52	52	8	4		85	712	58	44	50	44
	86		31	25	50	35	4		82	699	58	44	50	44
	83		29	33	49	27	4		79	682	58	44	50	44
	80	30	28	37	47	23	4	30	76	666	59	14	50	44
9—	77	30	27	14	45	46	4	30	73	648	59	14	50	44
	75		25	50	44	10	5		70	630	59	44	49	54
	72		24	36	42	24	5		67	618	59	44	49	44
	69	30	23	43	40	17	5	30	64	606	60	14	49	14
10—	67	30	22		39		6	30	61	563	61	14	48	14
	65		20	48	37	12	7		58	544	61	44	47	44
	62	30	19	35	35	25	7	30	55	525	62	14	47	14
	60		18	6	33	54	8		52	510	62	44	46	44
	58		17	13	31	47	9		49	481	63	44	45	44
11—	56		16	5	29	55	10		46	447	64	44	44	44
	55		14	56	28	4	12		43	405	66	44	42	44
	54		13	48	26	12	14		40	473	69	14	40	14

TABLE XIX.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 50 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	54	44
	176		87	15	88	45	0		176	998	54	44	54	44
	173		85	17	87	43	0		173	996	54	44	54	44
	170	30	83	21	86	39	0	30	170	994	55	14	54	14
1—	167	30	81	20	85	40	0	30	167	992	55	14	54	14
	164	30	79	20	84	40	0	30	164	984	55	14	54	14
	161	30	77	22	83	38	0	30	161	980	55	14	54	14
	159		75	25	82	35	1		158	976	55	44	53	44
2—	156		73	26	81	34	1		155	974	55	44	53	44
	153		71	26	80	34	1		152	970	55	44	53	44
	150		69	32	79	28	1		149	968	55	44	53	44
3—	147	30	67	38	78	22	1	30	146	958	56	14	53	14
	144	30	65	35	77	25	1	30	143	948	56	14	53	14
	141	30	62	40	77	11	1	30	140	939	56	14	53	14
	138	30	61	55	75	5	1	30	137	929	56	14	53	14
4—	136		60	3	73	57	2		134	920	56	44	52	44
	133		58	11	72	49	2		131	910	56	44	52	44
	130		56	35	71	25	2		128	901	56	44	52	44
	127		54	34	70	26	2		125	890	56	44	52	44
5—	124	30	52	47	69	13	2	30	122	881	57	14	52	14
	121	30	51	2	67	58	2	30	119	872	57	14	52	14
	118	30	48	40	67	20	2	30	116	863	57	14	52	14
	115	30	46	51	66	9	2	30	113	852	57	14	52	14
6—	113		45	54	64	6	3		110	840	57	44	51	44
	110		44	10	62	50	3		107	828	57	44	51	44
	107		42	31	61	29	3		104	816	57	44	51	44
	104	30	41	23	59	37	3	30	101	789	58	14	51	14
7—	101	30	39	17	58	43	3	30	98	778	58	14	51	14
	99		37	44	57	16	4		95	767	58	44	50	44
	96		35	59	56	1	4		92	756	58	44	50	44

TABLE XIX.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 50 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	94		35	16	54	44	4		90	745	58	44	50	44
	92	30	34	33	53	27	4	30	88	735	59	14	50	14
8—	89	30	32	52	52	8	4	30	85	725	59	14	50	14
	87		31	25	50	35	5		82	694	59	44	49	44
	84		29	33	49	27	5		79	681	59	44	49	44
	81	30	28	37	47	23	5	30	76	669	60	14	49	14
9—	79		27	14	45	46	6		73	646	60	44	48	44
	76	30	25	50	44	10	6	30	70	610	61	14	48	14
	74		24	36	42	24	7		67	604	61	44	47	44
	71	30	23	43	40	17	7	30	64	586	62	14	47	14
	69		22		39		8		61	569	62	44	46	44
10—	67		20	48	37	12	9		58	538	63	44	45	44
	64	30	19	35	35	25	9	30	55	523	64	14	45	44
	63		18	6	33	54	11		52	487	65	44	43	44
	61		17	13	31	47	12		49	464	66	44	42	44
	59	30	16	5	29	55	13	30	46	436	68	14	41	14

TABLE XX.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 55 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe. and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M	D,	M	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	54	44
	176		87	15	88	45	0		176	999	54	44	54	44
	173		85	17	87	43	0		173	998	54	44	54	44
1—	170		83	21	86	39	0	30	170	996	55	14	54	14
	167		81	20	85	40	0	30	167	995	55	14	54	14
	164		79	20	84	40	0	30	164	991	55	14	54	14
	162		77	22	83	38	1		161	987	55	44	53	44
2—	159		75	25	82	35	1		158	983	55	44	53	44
	156		73	26	81	34	1		155	979	55	44	53	44
	153		71	26	80	34	1		152	971	55	44	53	44
	150		69	32	79	28	1		149	964	55	44	53	44
3—	147	30	67	38	78	22	1	30	146	957	56	14	53	14
	144	30	65	35	77	25	1	30	143	951	56	14	53	14
	142		62	40	77	11	2		140	943	56	44	52	44
	139		61	55	75	5	2		137	935	56	44	52	44
4—	136		60	3	73	57	2		134	928	56	44	52	44
	133	30	58	11	72	49	2	30	131	918	57	14	52	14
	130	30	56	35	71	25	2	30	128	908	57	14	52	14
	127	30	54	34	70	26	2	30	125	898	57	14	52	14
5—	125		52	47	69	13	3		122	883	57	44	51	44
	122		51	2	67	58	3		119	868	57	44	51	44
	119		48	40	67	20	3		116	857	57	44	51	44
	116	30	46	51	66	9	3	30	113	846	58	14	51	14
6—	113	30	45	54	64	6	3	30	110	835	58	14	51	14
	111		44	10	62	50	4		107	820	58	44	50	44
	108		42	31	61	29	4		104	804	58	44	50	44
	105	30	41	23	59	37	4	30	101	795	59	14	50	14
7—	102	30	39	17	58	43	4	30	98	785	59	14	50	14
	99	30	37	44	57	16	4	30	95	775	59	14	50	14
	97		35	59	56	1	5		92	749	59	44	49	44

TABLE XX.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 55 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe. and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	95		35	16	54	44	5		90	741	59	44	49	44
8—	93	30	34	33	53	27	5	30	88	733	60	14	49	14
	90	30	32	52	52	8	5	30	85	725	60	14	49	14
	88		31	25	50	35	6		82	699	60	44	48	44
	85	30	29	33	49	27	6	30	79	678	61	14	48	14
	83		28	37	47	23	7		76	658	61	44	47	44
	80	30	27	14	45	46	7	30	73	639	62	14	47	14
9—	78		25	50	44	10	8		70	620	62	44	46	44
	75	30	24	36	42	24	8	30	67	606	63	14	46	14
	73		23	43	40	17	9		64	588	63	44	45	44
	71		22		39		10		61	560	64	44	44	44
	69		20	48	37	12	11		58	535	65	44	43	44
10—	67	30	19	35	35	25	12	30	55	502	67	14	42	14
	66		18	6	33	54	14		52	474	68	44	40	44
	65		17	13	31	47	16		49	441	70	44	39	44
	64		16	5	29	55	18		46	416	72	44	37	44

TABLE XXI.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 60 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe. and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M	D.	M.	D.		D.	M.	D.	M.
	180		90		90		0		180	1000	54	44	54	44
	176		87	15	88	45	0		176	999	54	44	54	44
	173		85	17	87	43	0		173	998	54	44	54	44
1—	170		83	21	86	39	0	30	170	997	55	14	54	14
	167		81	20	85	40	0	30	167	995	55	14	54	14
	164	30	79	20	84	40	0	30	164	992	55	14	54	14
	162		77	22	83	38	1		161	989	55	44	53	44
2—	159		75	25	82	35	1		158	986	55	44	53	44
	156		73	26	81	34	1		155	981	55	44	53	44
	153		71	26	80	34	1		152	975	55	44	53	44
	150	30	69	32	79	28	1	30	149	969	56	14	53	14
3—	147	30	67	38	78	22	1	30	146	958	56	14	53	14
	145		65	35	77	25	2		143	948	56	44	52	44
	142		62	40	77	11	2		140	939	56	44	52	44
	139		61	55	75	5	2		137	931	56	44	52	44
4—	136	30	60	3	73	57	2	30	134	923	57	14	52	14
	133	30	58	11	72	49	2	30	131	915	57	14	52	14
	130	30	56	35	71	25	2	30	128	907	57	14	52	14
	128		54	34	70	26	3		125	900	57	44	51	44
5—	125		52	47	69	13	3		122	885	57	44	51	44
	122	30	51	2	67	58	3	30	119	870	58	14	51	14
	119	30	48	40	67	20	3	30	116	857	58	14	51	14
	117		46	51	66	9	4		113	844	58	44	50	44
6—	114		45	54	64	6	4		110	830	58	44	50	44
	111	30	44	10	62	50	4	30	107	817	59	14	50	14
	108	30	42	31	61	29	4	30	104	806	50	14	50	14
	106		41	23	59	37	5		101	794	59	44	49	44
7—	103		39	17	58	43	5		98	782	59	44	49	44
	100	30	37	44	57	16	5	30	95	770	60	14	49	14

TABLE XXI.

A Table for the moſt advantageous working of Ships whoſe curve-lined Prow is 60 Degrees.

Points of the Compaſs.	The Angles made by the Wind's and Ship's Courſe.		The beſt Angles for the Sails and Keel.		The beſt Angles for the Sails and Wind.		The Lee-ways, or Angles made with the Ship's Courſe and Keel.		The Angles of the Wind and Keel.	The Ratio of the Velocities.	The beſt Angles of the Rudder to tack about.		The beſt Angles of the Rudder for veering the Ship.	
	D.	M.	D.	M.	D.	M.	D.	M.	D.		D.	M.	D.	M.
	98		35	59	56	1	6		92	754	60	44	48	44
	96		35	16	54	44	6		90	749	60	44	48	44
	94	30	34	33	53	27	6	30	88	727	61	14	48	14
	92		32	52	52	8	7		85	707	61	44	47	44
8—	89	30	31	25	50	35	7	30	82	688	62	44	47	14
	87		29	33	49	27	8		79	670	62	44	46	44
	84	30	28	37	47	23	8	30	76	654	63	14	46	14
	82		27	14	45	46	9		73	636	63	44	45	44
9—	80		25	50	44	10	10		70	609	64	44	44	44
	77	30	24	36	42	24	10	30	67	595	65	14	44	14
	75	30	23	43	40	17	11	30	64	571	66	14	43	14
	73	30	22		39		12	30	61	548	67	14	42	14
	72		20	48	37	12	14	30	58	518	69	14	40	14
	71		19	35	35	25	16		55	485	70	44	37	44
	70		18	6	33	54	18		52	455	72	44	36	44
	69		17	13	31	47	20		49	429	74	44	34	44

Since we have Occaſion in the Reſolution of the greateſt Part of the Problems of the working of Ships, for the Sines and Tangents of Angles for the fourth Problem; we have thought fit here to ſubjoin a ſmall Table of the Sines and Tangents of every Degree, for the Convenience of thoſe who may uſe our Tables in the Practice of Navigation.

D.	Sines.	Tangents.	D.	Sines.	Tangents.
1	1145	1746	28	46947	53171
2	3490	3492	29	48481	55431
3	5234	5241	30	50000	57735
4	6976	6993	31	51504	50086
5	8716	8749	32	52992	62487
6	10453	10510	33	54464	64941
7	12187	12278	34	55919	67451
8	13917	14054	35	57358	70021
9	15643	15838	36	58779	72654
10	17365	17633	37	60181	75355
11	19081	19439	38	61566	78129
12	20791	21256	39	62932	80978
13	22495	23087	40	64279	83910
14	24192	24933	41	65606	86929
15	25882	26795	42	66913	90040
16	27564	28675	43	68200	93252
17	29237	30573	44	69466	96569
18	30902	32492	45	70711	100000
19	32557	34433	46	71934	103553
20	34202	36397	47	73135	107237
21	35837	38386	48	74314	111061
22	37461	40403	49	75471	115037
23	39073	42447	50	76604	119175
24	40674	44523	51	77715	123490
25	42262	46631	52	78801	127994
26	43837	48773	53	79864	132704
27	45399	50953	54	80902	137638

Table of Sines and Tangents.

D.	Sines.	Tangents.	D.	Sines.	Tangents.
54	80902	137638	72	95106	307768
55	81915	142815	73	95630	327085
56	82904	148256	74	96126	348741
57	83867	153986	75	96593	373205
58	84805	160033	76	97030	401078
59	85717	165428	77	97437	433148
60	86603	173205	78	97815	470463
61	87462	180405	79	98163	514455
62	88295	188073	80	98481	567128
63	89101	196261	81	98769	631375
64	89879	205030	82	99027	711537
65	90631	214451	83	99255	814435
66	91355	224604	84	99452	951436
67	92050	235585	85	99619	1143005
68	92718	247509	86	99756	1430067
69	93358	260509	87	99863	1908114
70	93969	274748	88	99939	2863624
71	94552	290421	89	99985	5728996
			90	100000	

The END of the Tables.

BOOKS printed for C. Davis, *againſt* Gray's-Inn-Gate *in* Holbourn.

1. The Figure of the Earth determined, from Obſervations made by Order of the *French* King, at the Polar Circle: by Meſſ. *De Maupertuis*, *Camus*, *Clairaut*, *De Monnier*, Members of the Royal Academy of Sciences, the Abbé *Outhier*, Correſpondent of the Academy, and Mr. *Celſius*, Profeſſor of Aſtronomy at *Upſal*. Tranſlated from the *French* of M. *De Maupertuis*.

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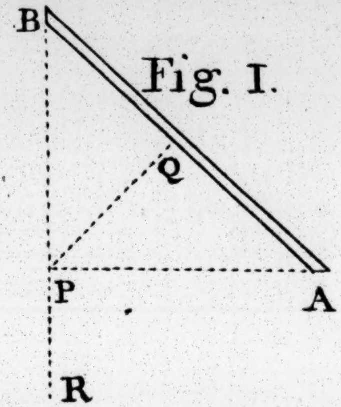

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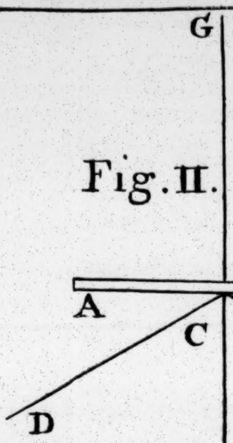

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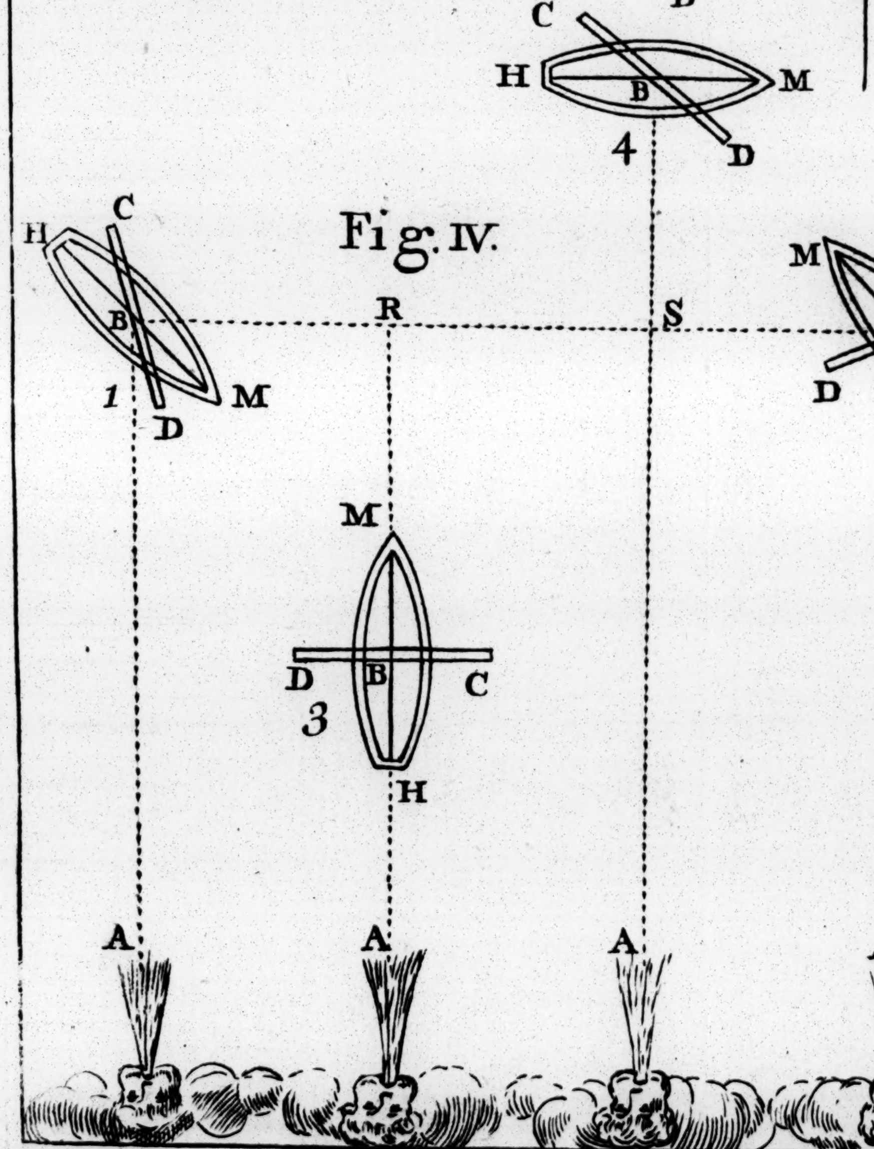

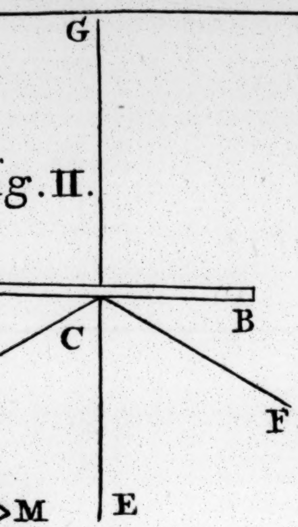
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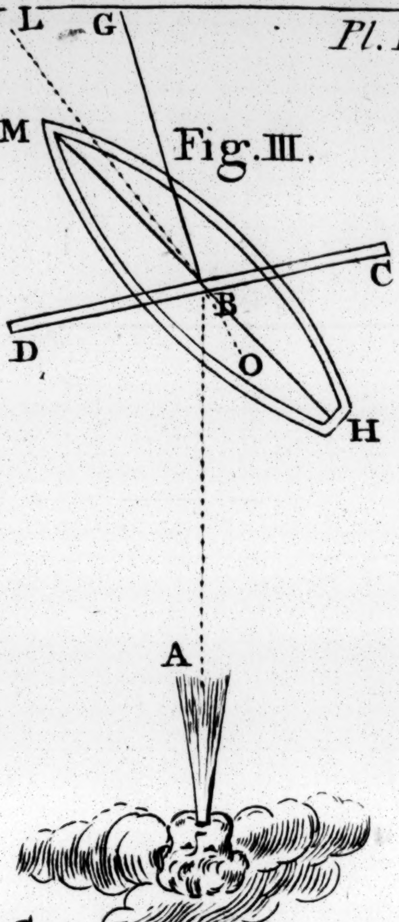
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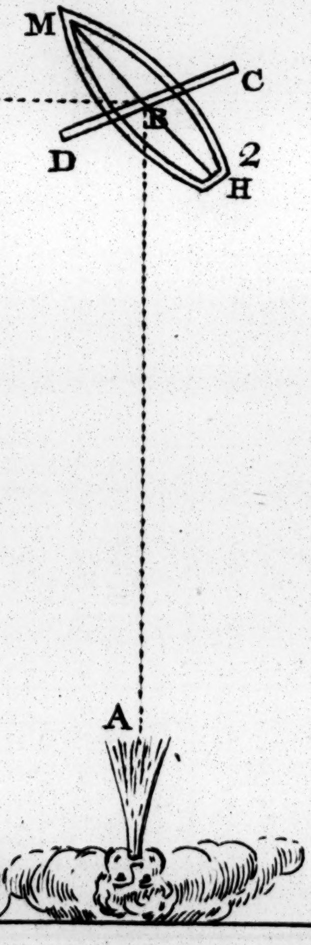

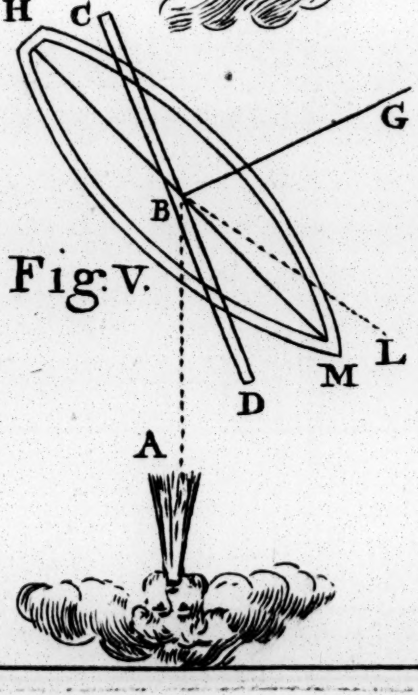
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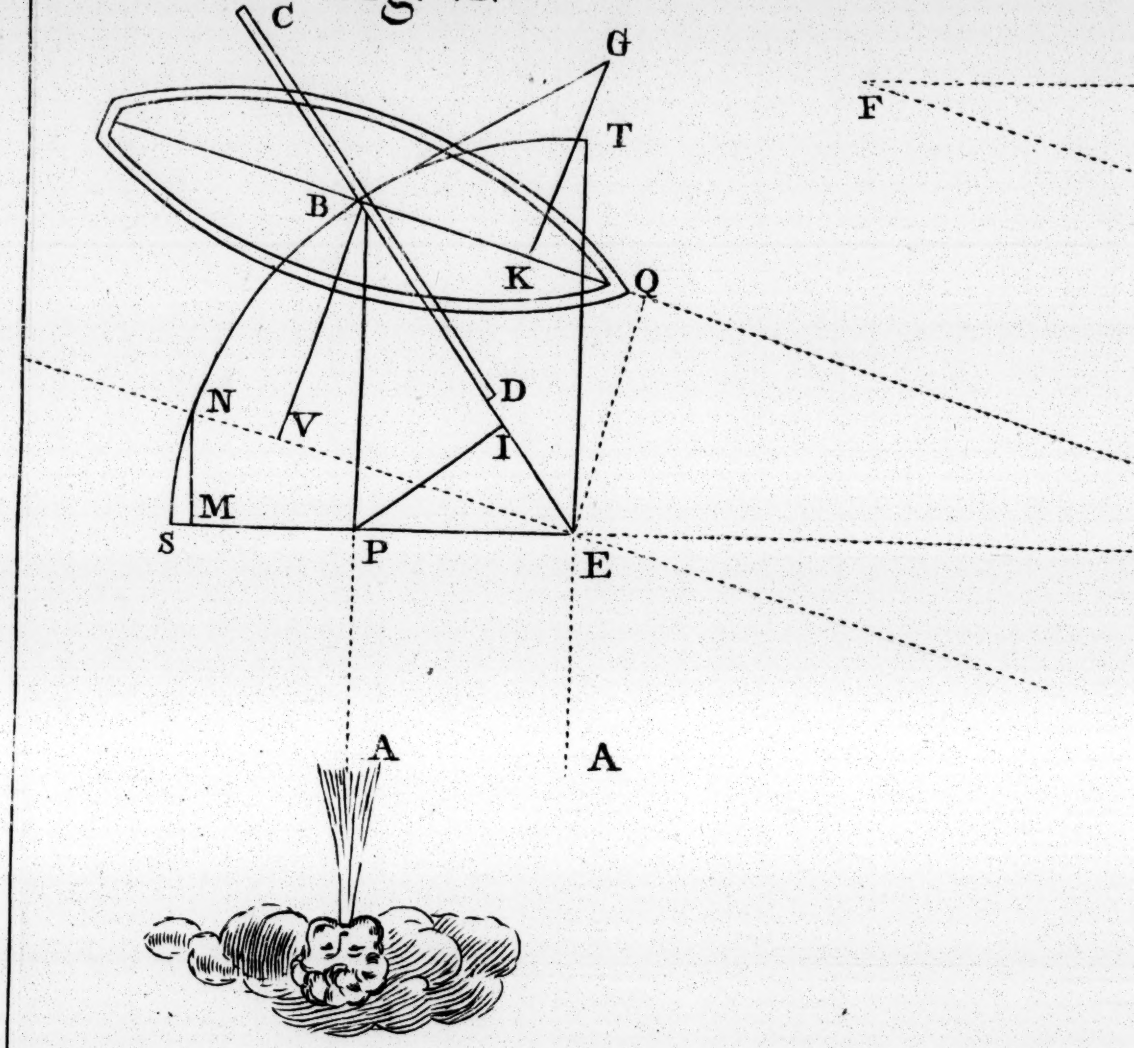

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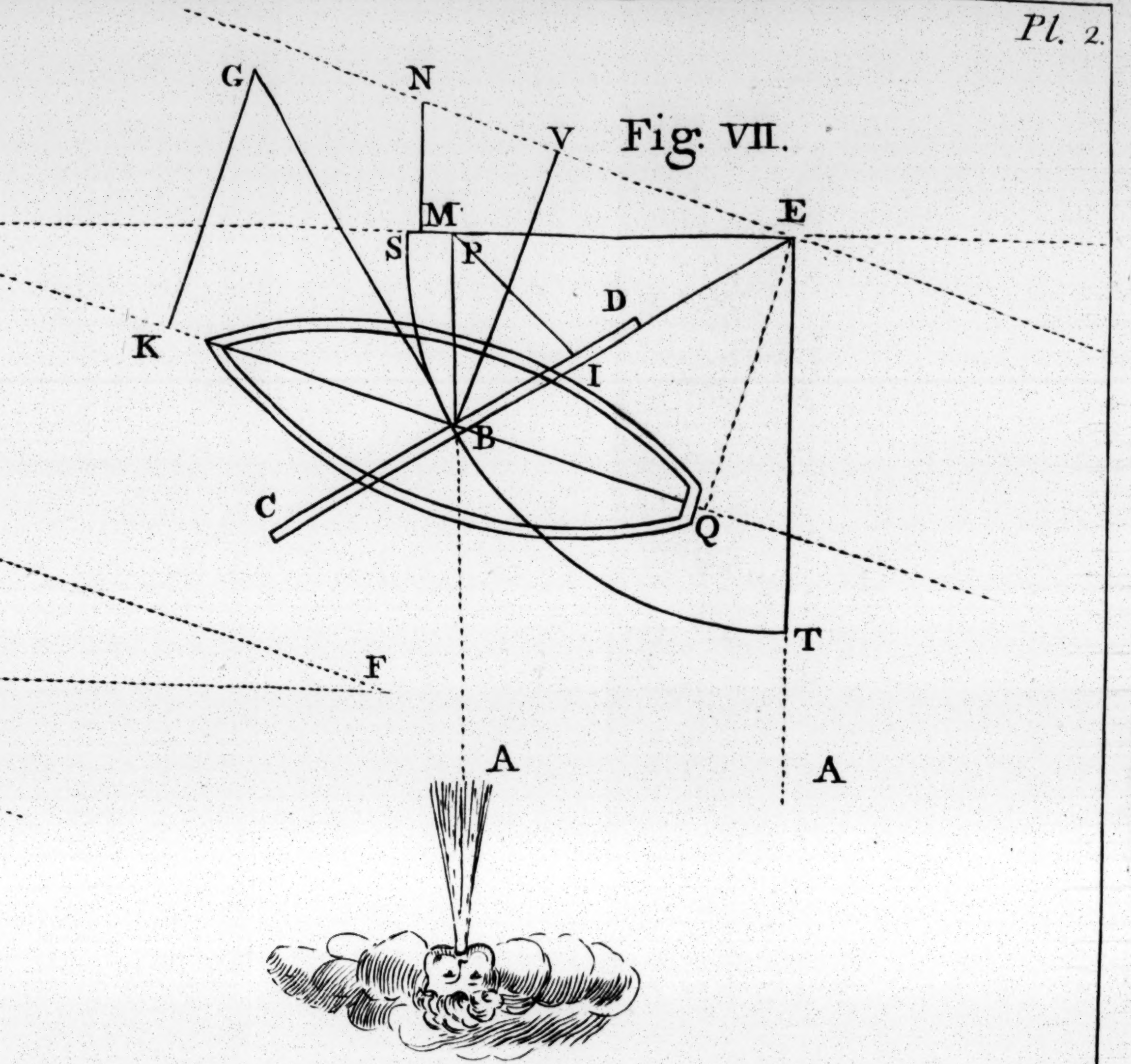

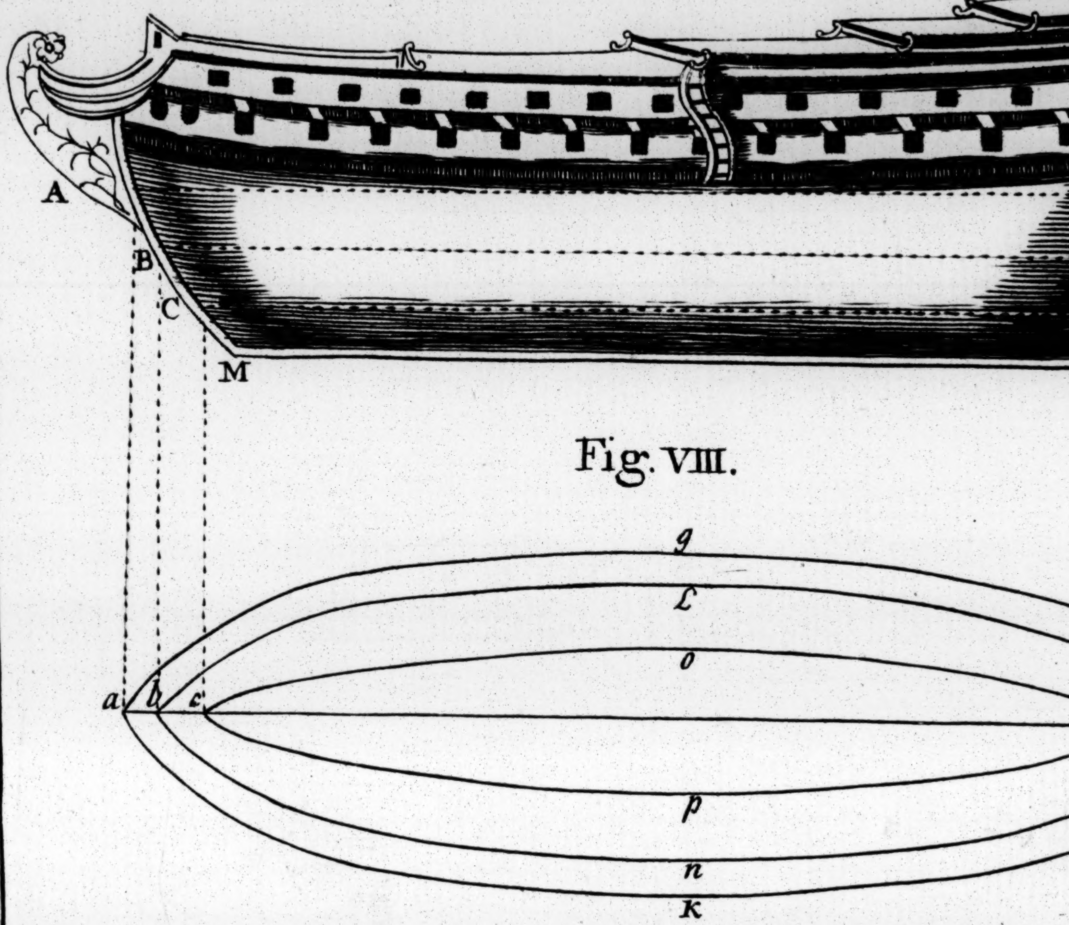

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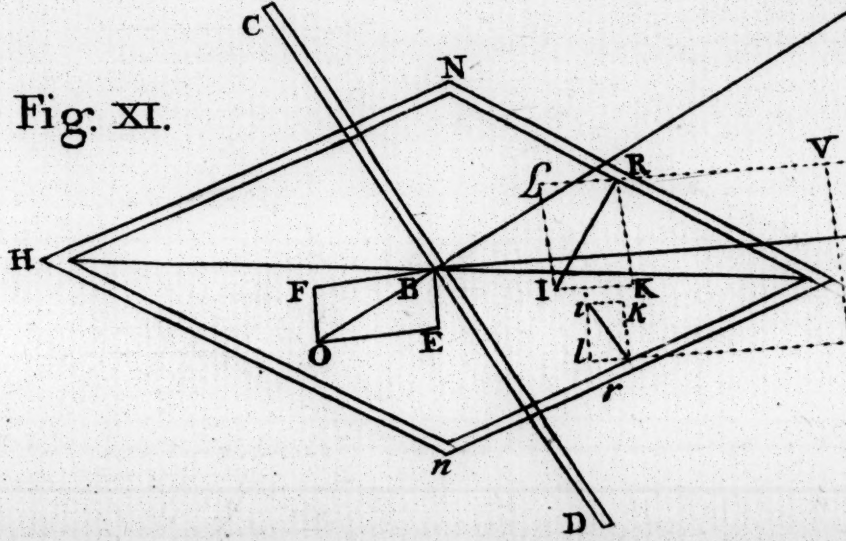

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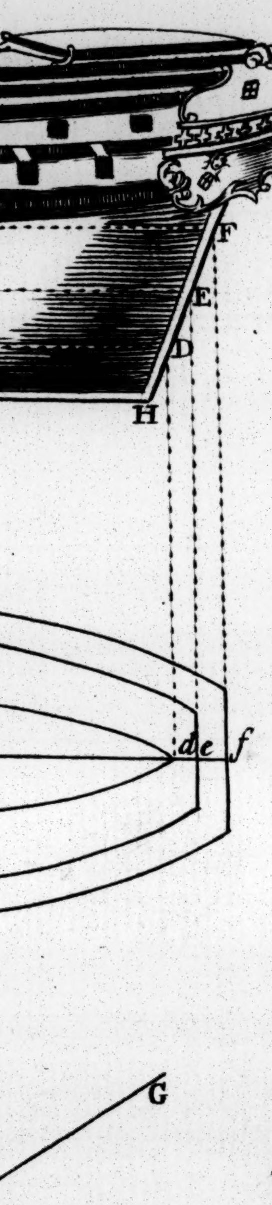

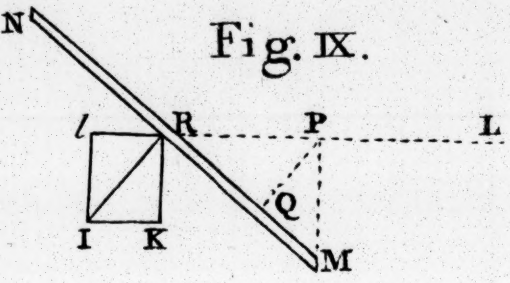
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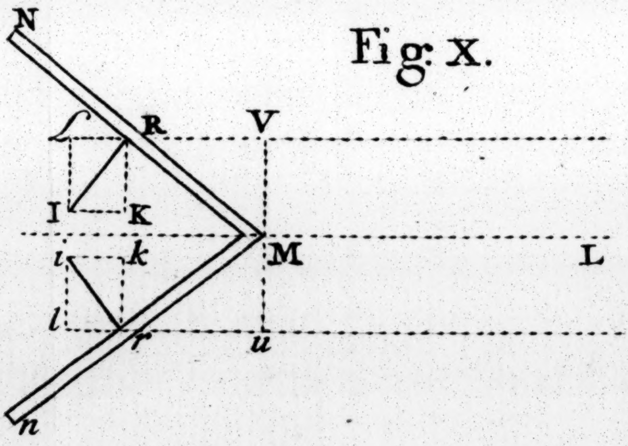
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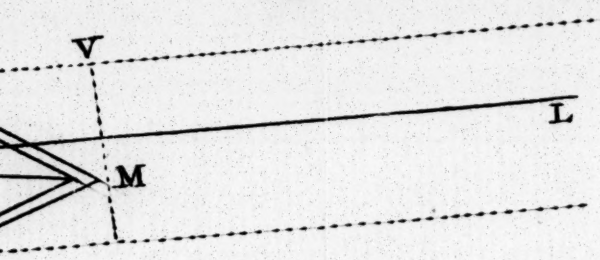

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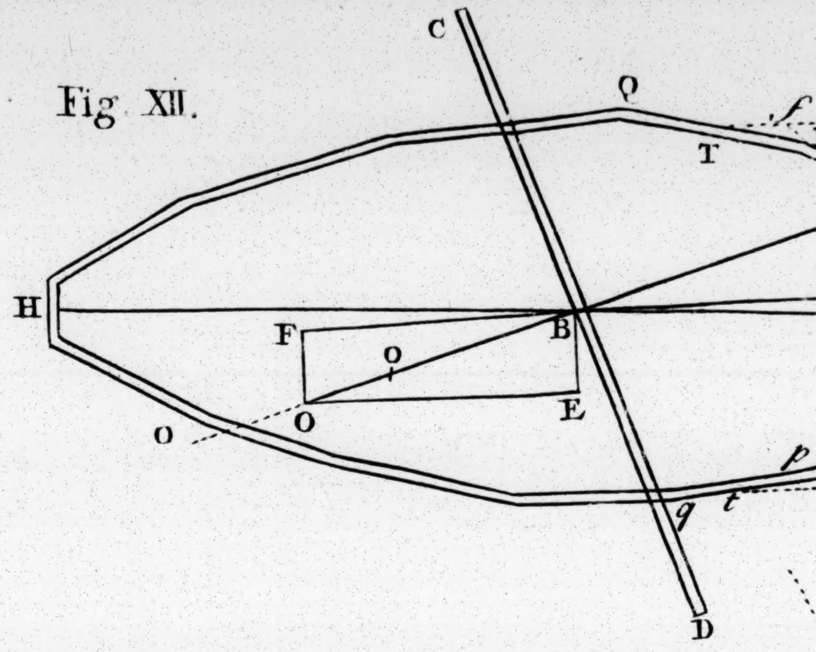

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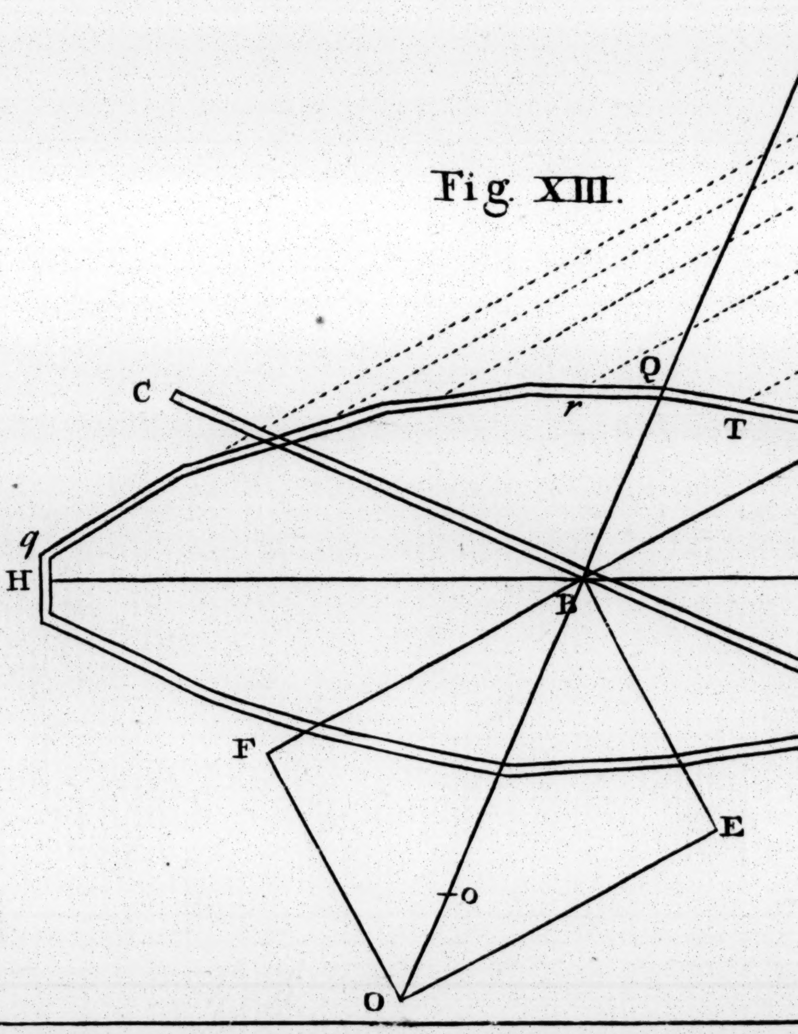

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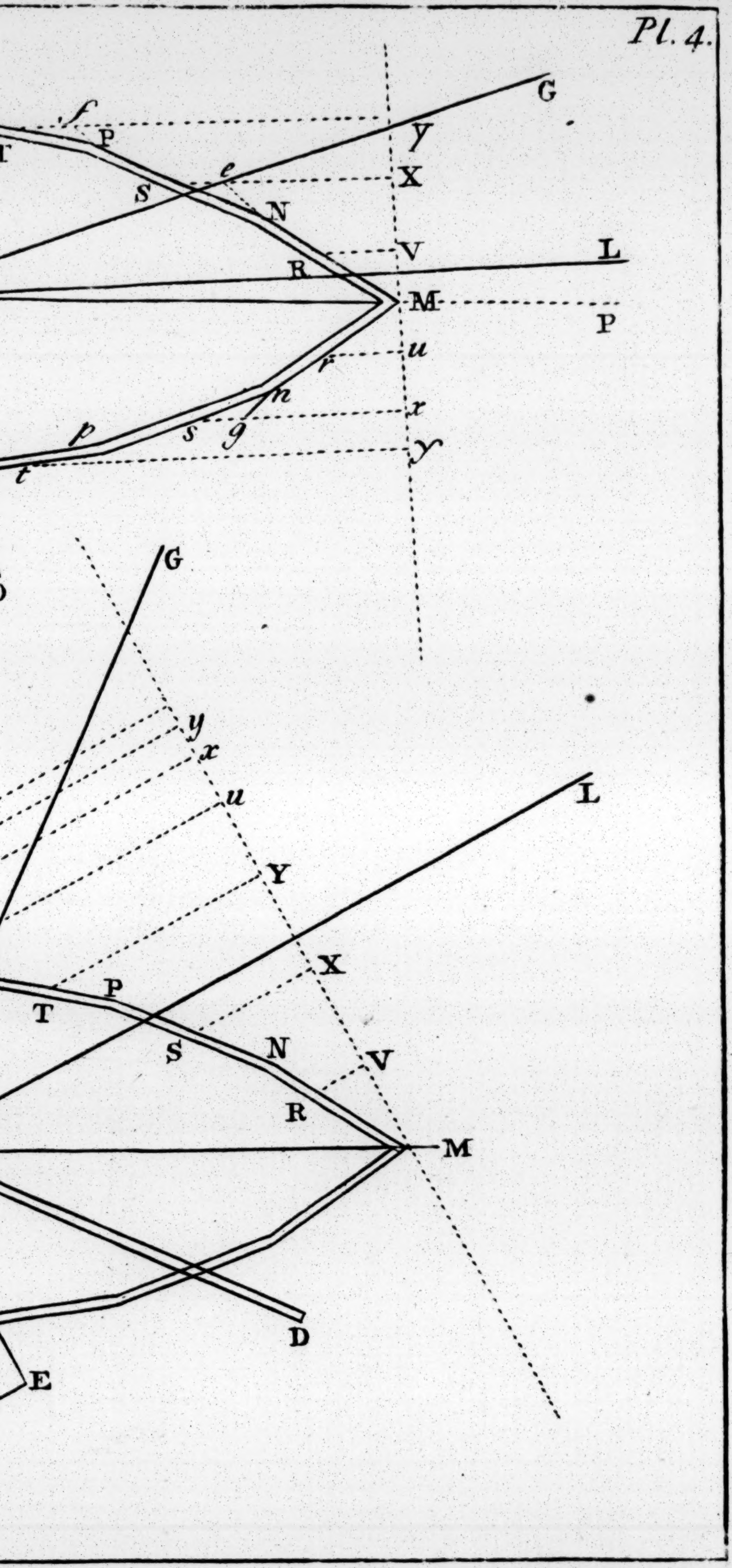

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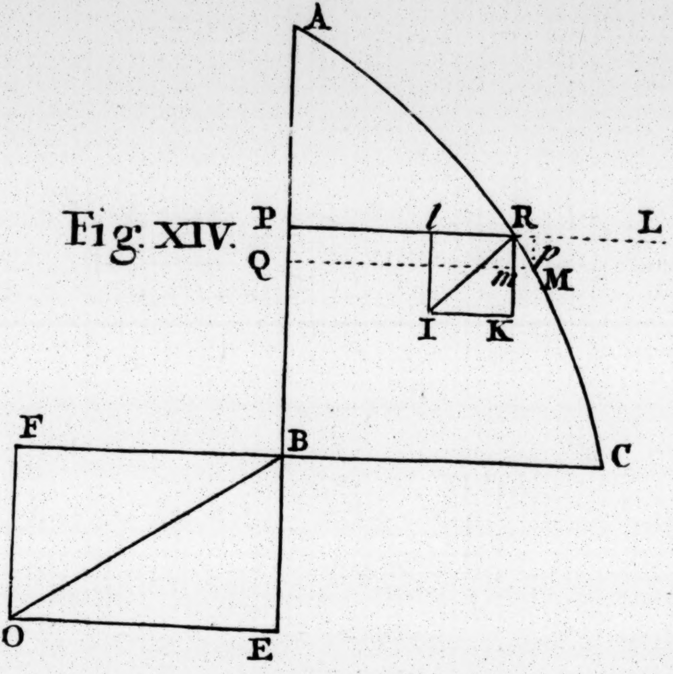

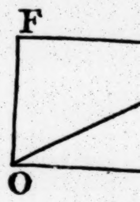

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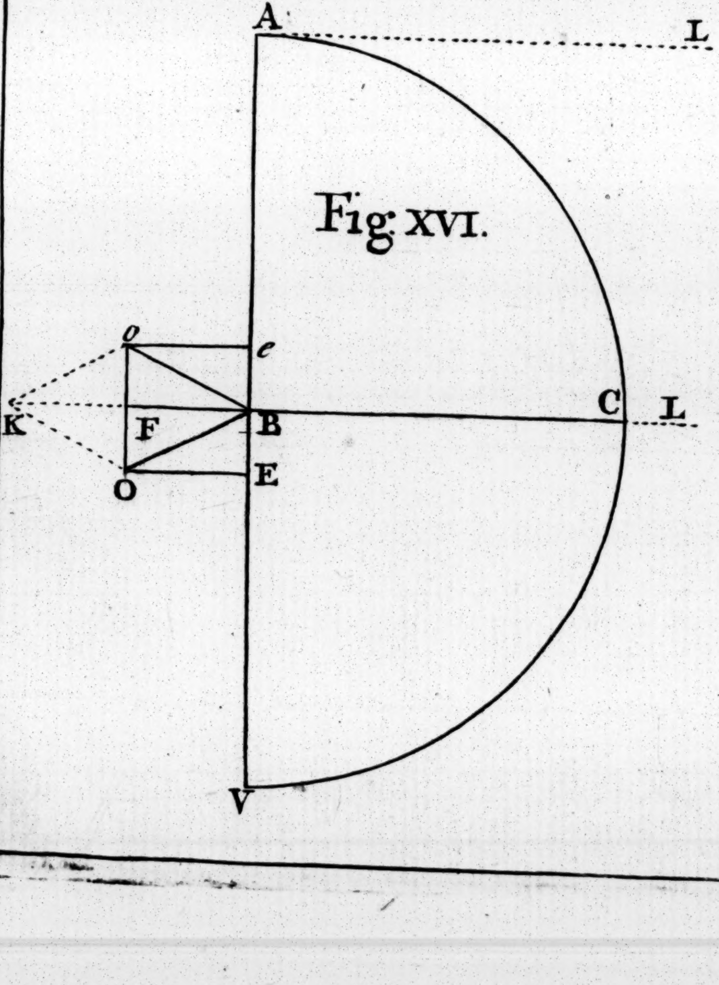

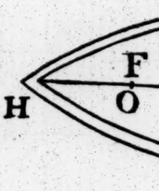

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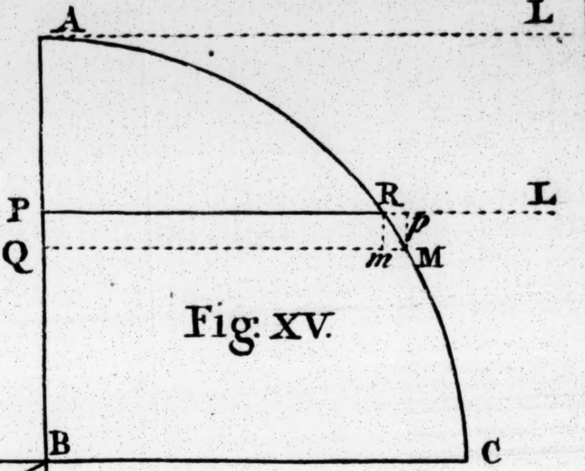

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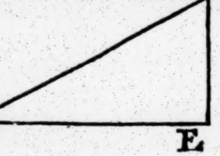

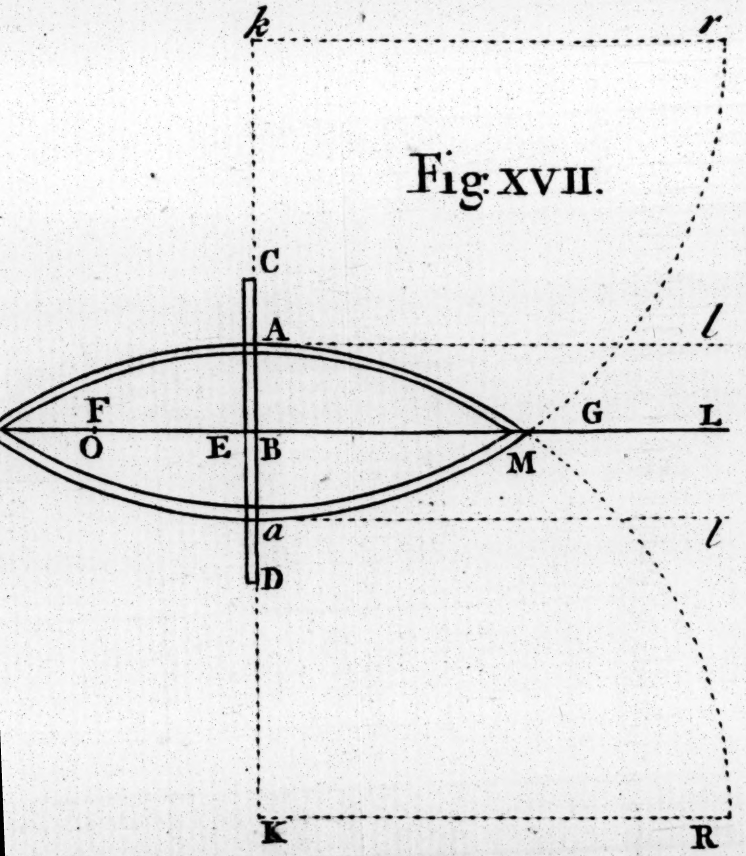

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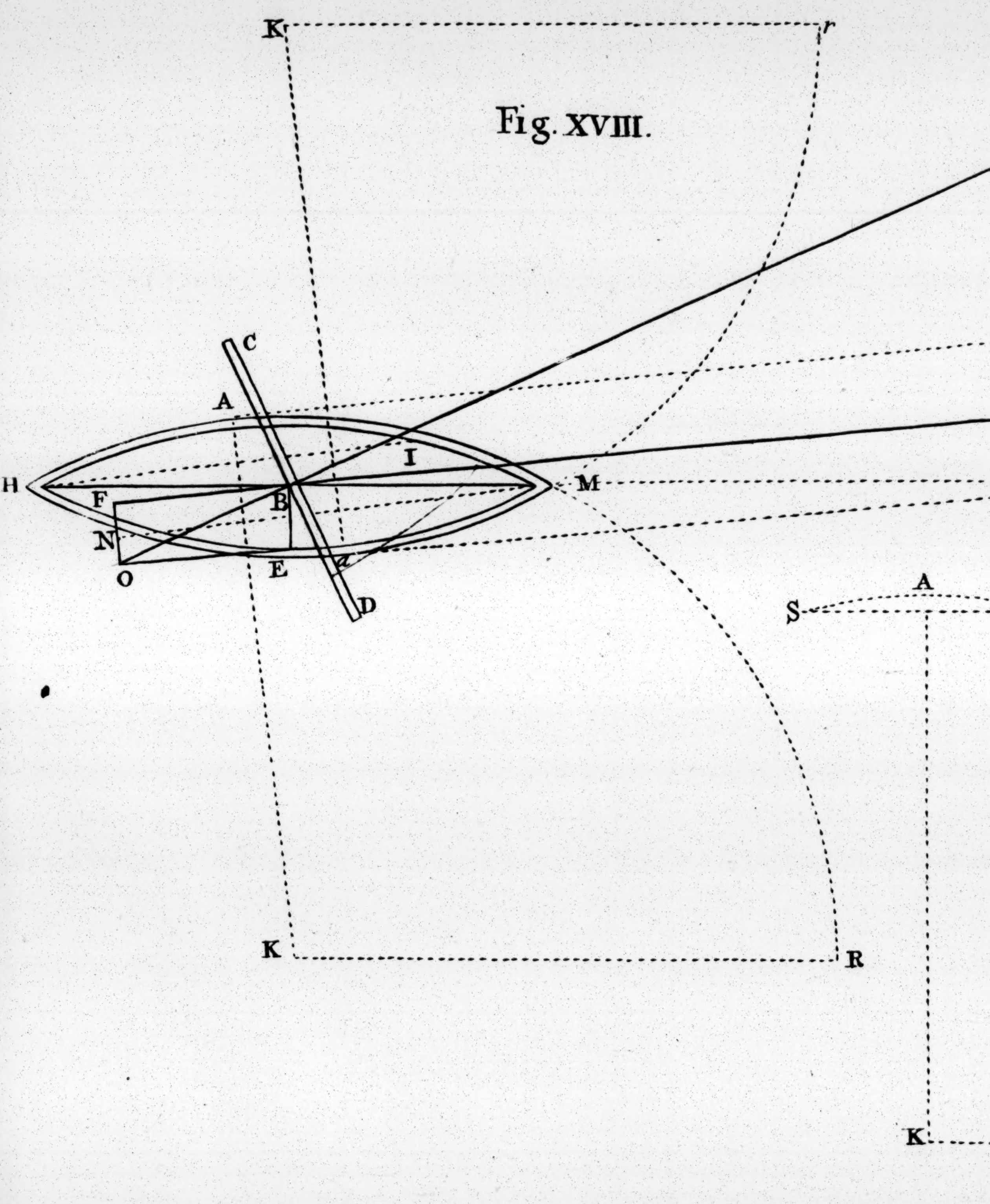
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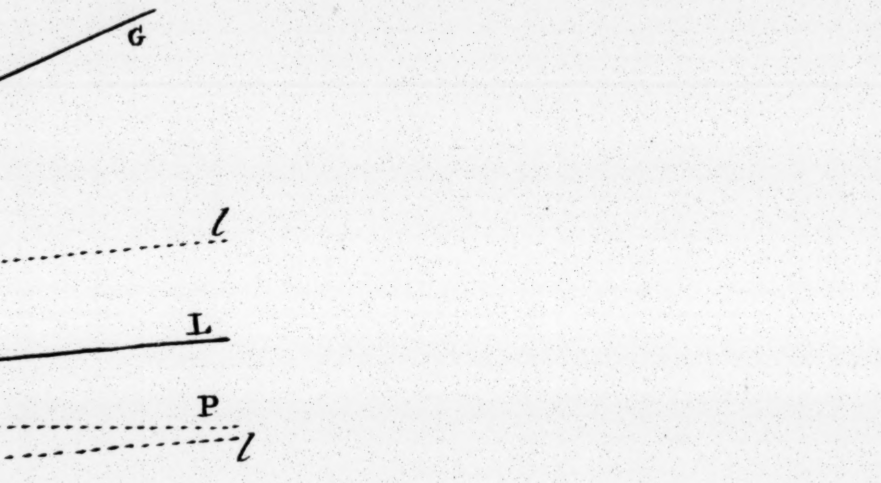

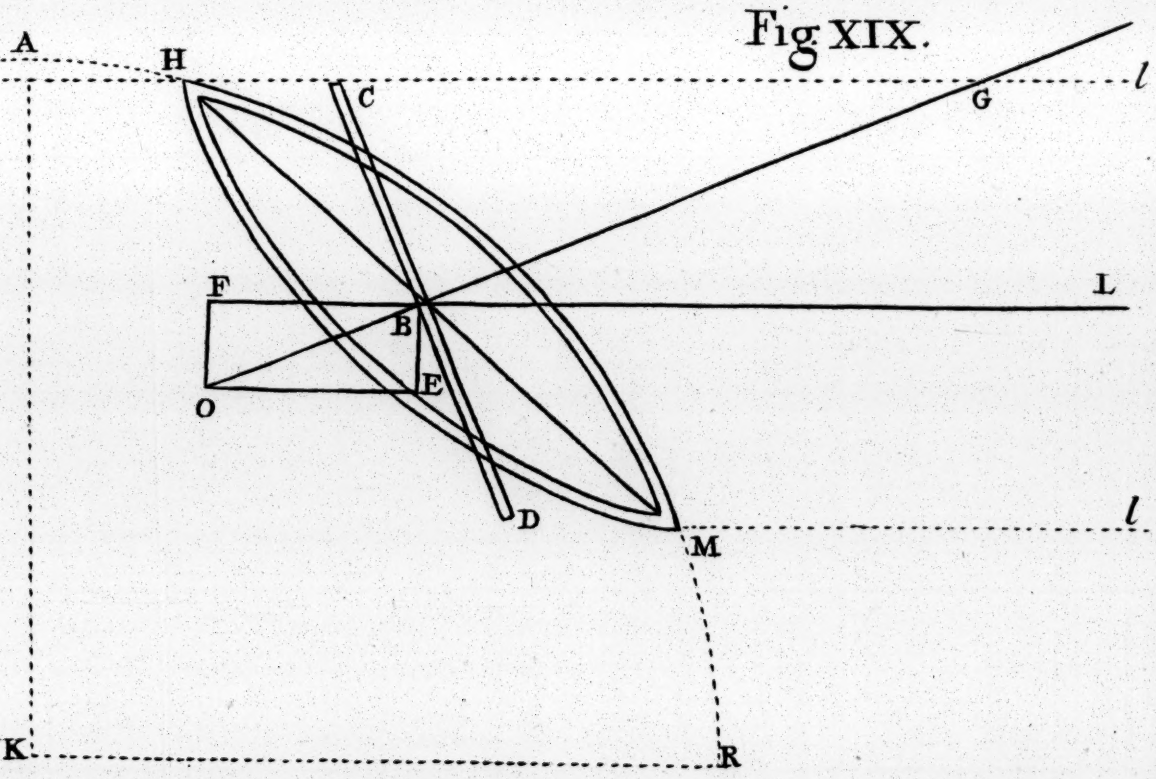

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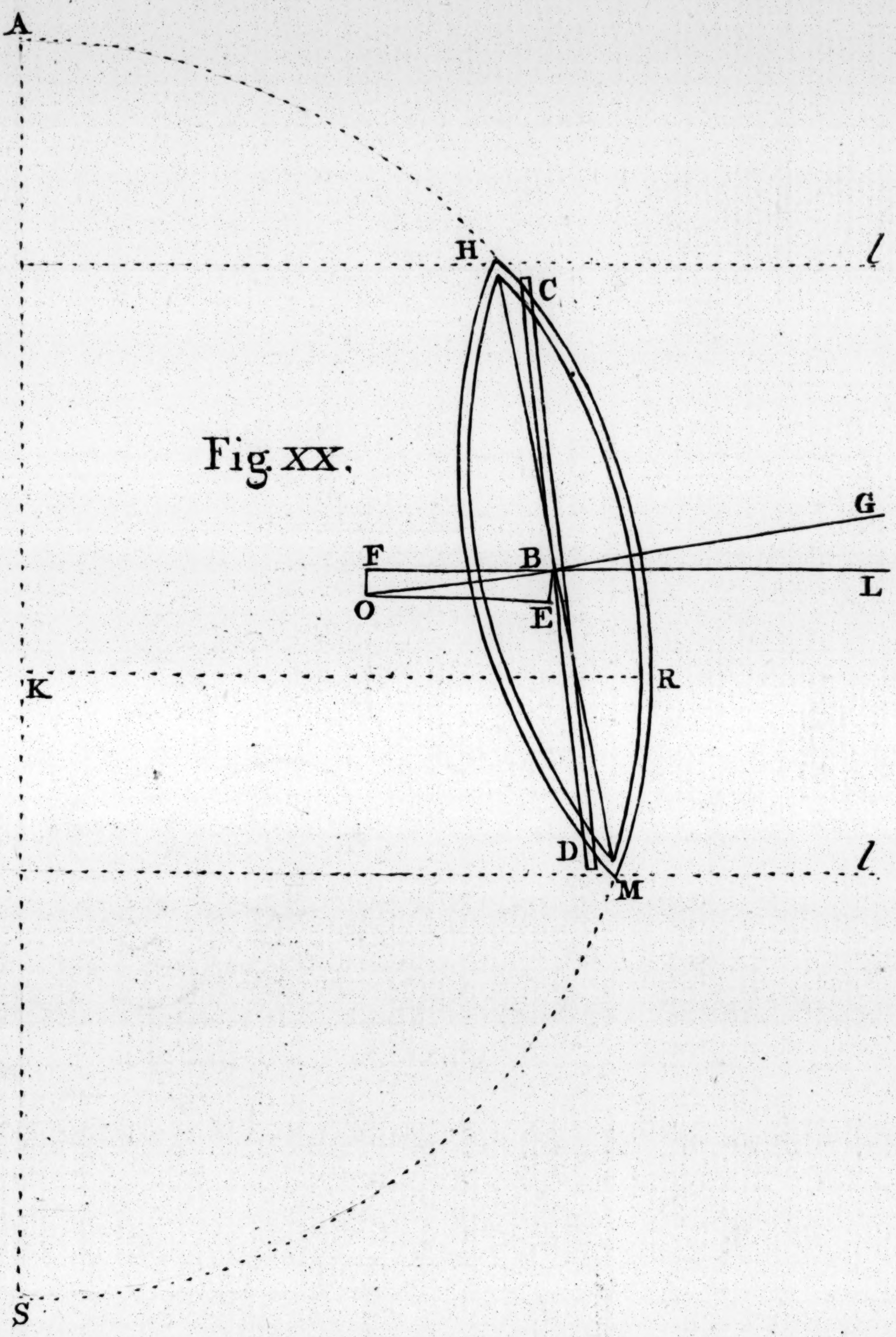

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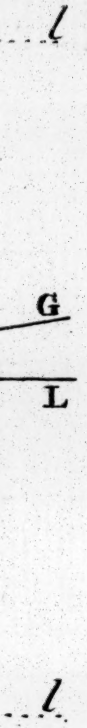

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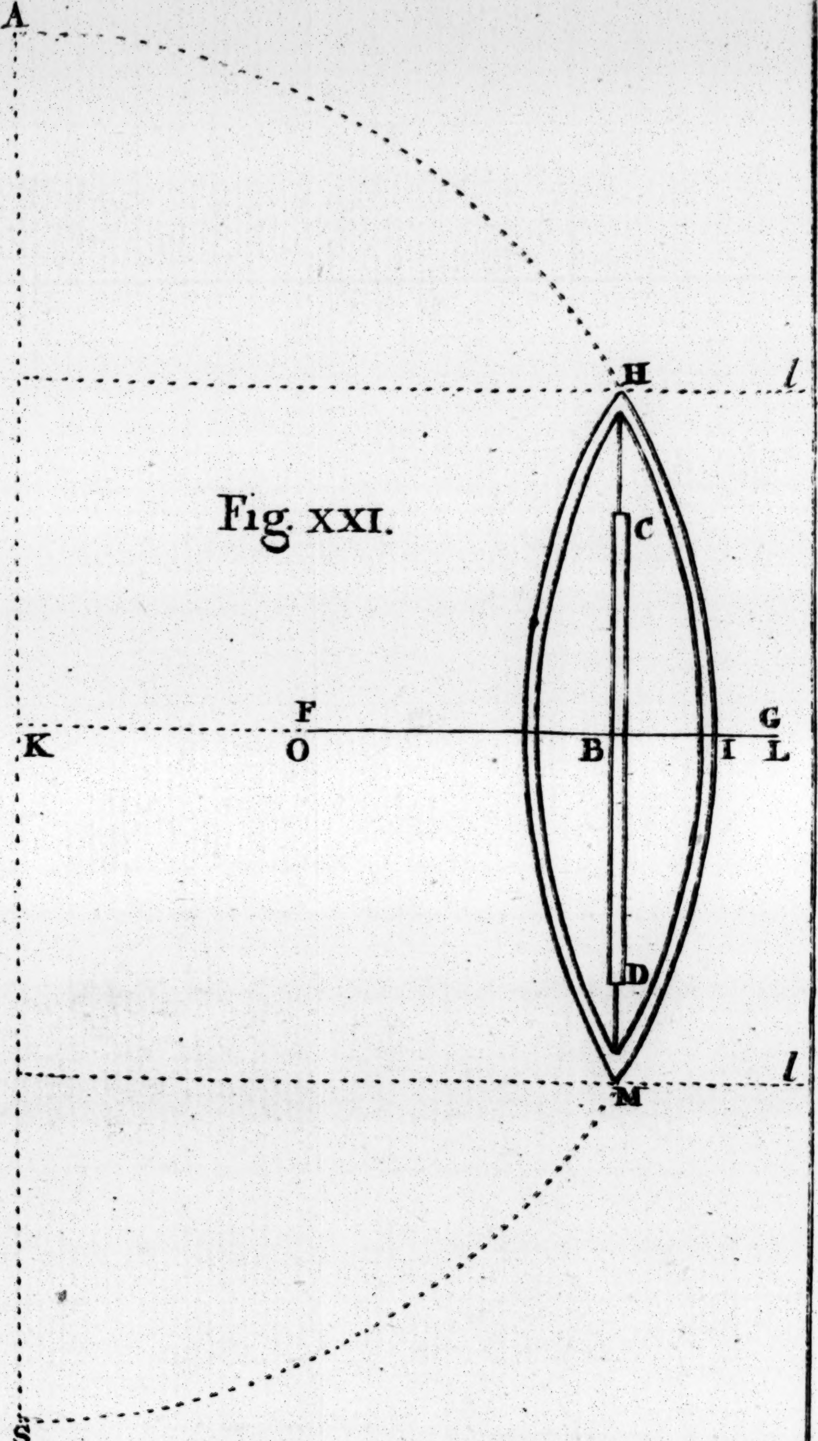

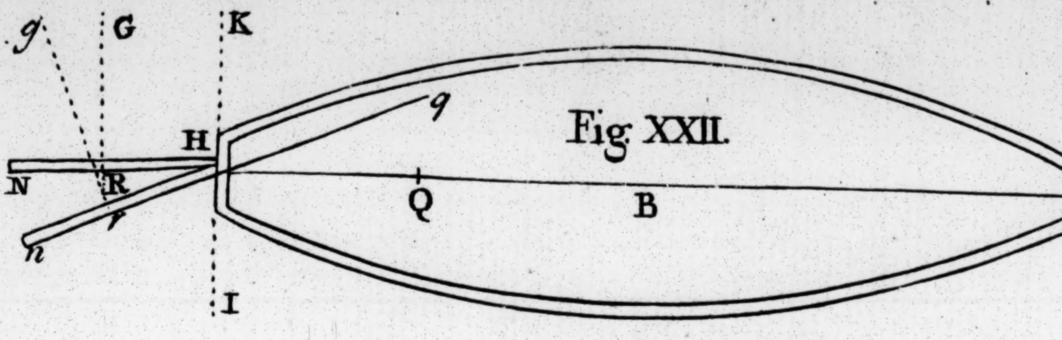

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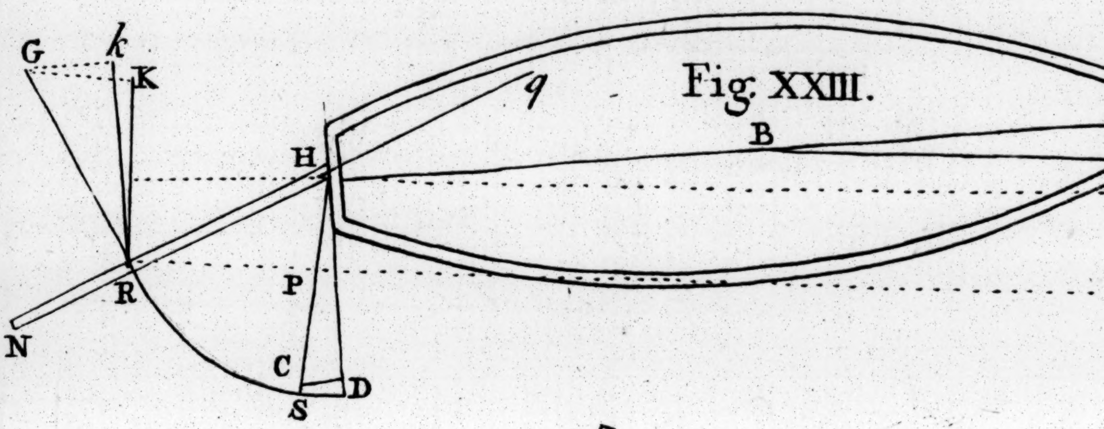

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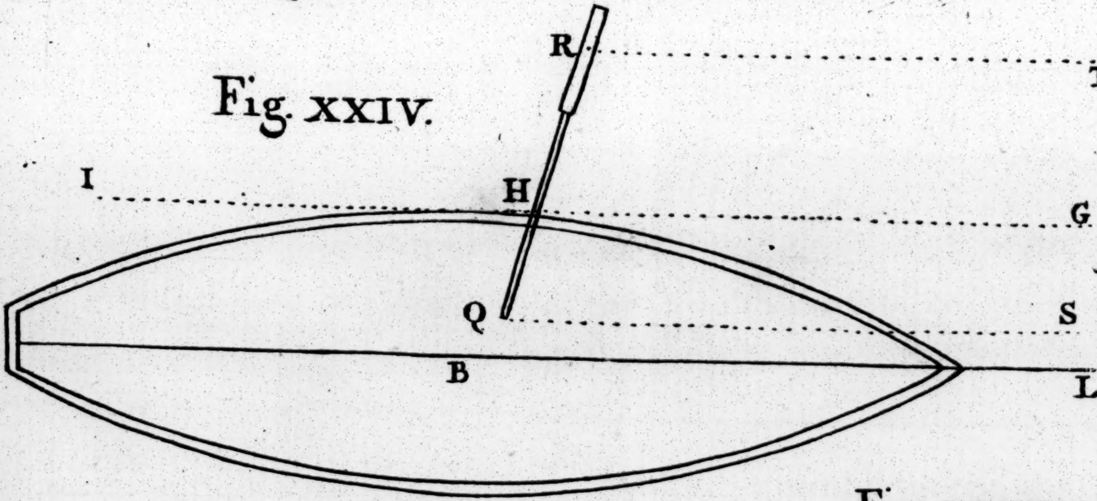

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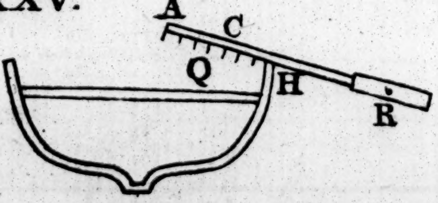

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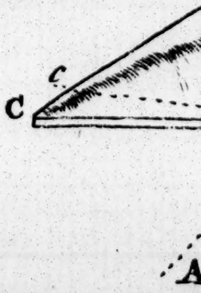

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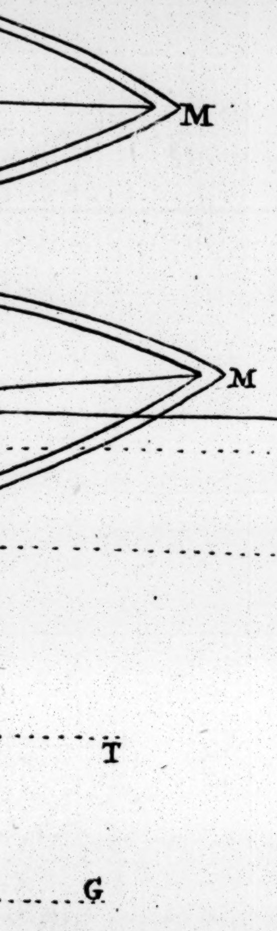

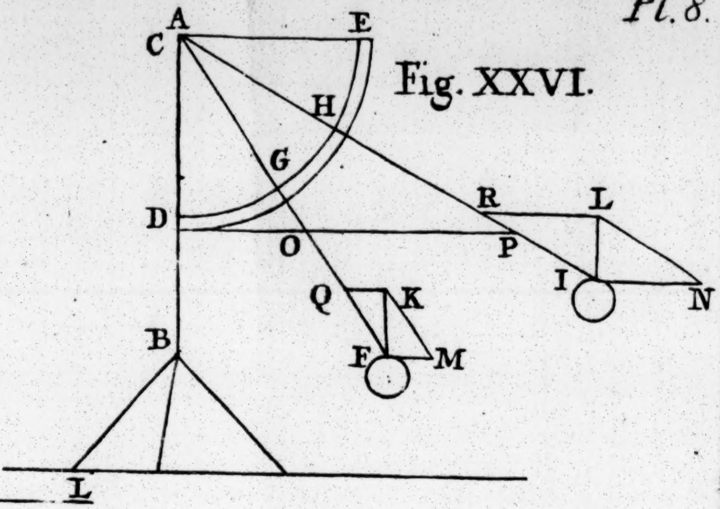

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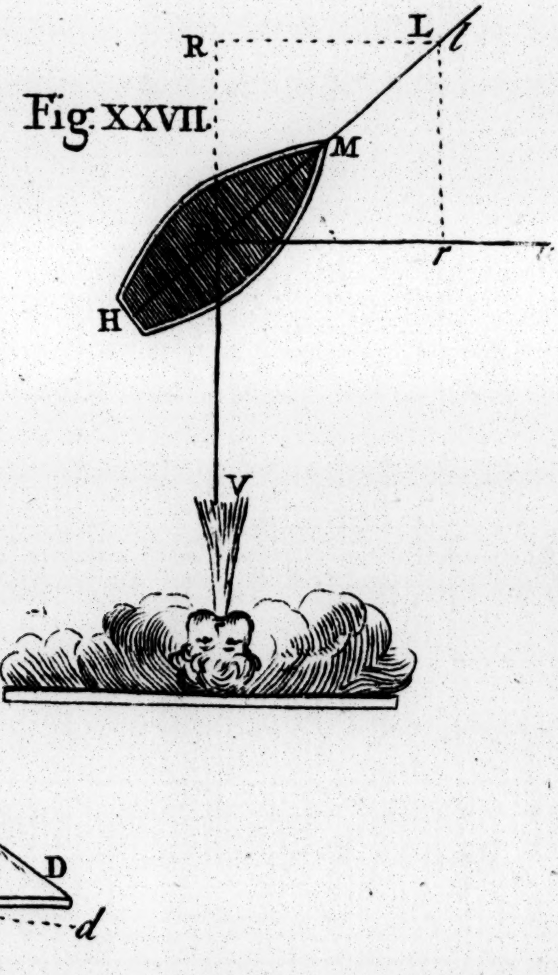

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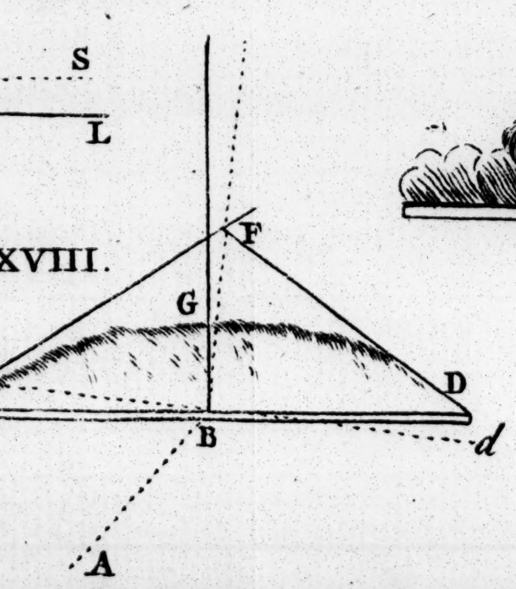

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